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MATHEMATICS IN LIFE

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ABSTRACT

In the work different fields of applications of mathematics are examined, namely: Mathematics and music, Mathematics and education, Mathematics and sport, Mathematics and modern technologies. The purpose of this work is to demonstrate on particular examples, the way mathematics has penetrated all spheres of human activities, in one form or another. The use of mathematics in sports is represented by the example of basketball (strategy and tactics of the game, calculation of results). Mathematics in music reveals itself in symmetry, euphony and the construction of scales. The application of mathematics in education is exemplified by the development of the schools of ancient Greece and Rome and schools of a later period. The relation of Mathematics with modern life is shown in the section "Mathematics and modern technologies".

It is difficult to name at least one kind of human activity which is not related to mathematics anyhow. Of course, it is impossible to disclose all the spheres of application of mathematics in one work, but the students used the analytical and research approach to the given task, and the corresponding conclusions were formulated by them.

CONTENTS

INTRODUCTION

Mathematics was developed to understand the cycles of nature as well as seasonal cycles. Ancient people understood the need to define time in relation to celestial movements for agricultural, astronomical and navigational reasons. They also understood that math is all around us, in everything we do.

MATHEMATICS IN LIFE

Sometimes we do not even notice how often we use math in our everyday routine. Math is the building block for everything in our daily life.

Where do people use mathematics?
Currency exchanges (divisions)
Travelling (budget, fuel needed for the car)
Architecture (calculations, geometry)
Weather (probability, numbers)
Bank transactions (so we would not be deceived)
Planning family budget (money-spending)
Money (it is counted in numbers)

Which professions use maths?
Shopkeeper(giving change)
Cooker (proportions)
Teacher
Doctor (amounts of medicine)
Accountant (calculating budget)
Banker
Engineer (calculations, geometry)

FUTURE IS MATH

Today, it doesn’t come as a surprise when you hear that future belongs to robots. Robots help people in routine problems, and math is the keystone of robotics. All commands executed by robots are in fact thousands of numbers and equations transformed by the computers into commands. Banking is something that we use every day. We also know about the FUTURE e-banking system called blockchain, where data is gathered and ordered into blocks, which are chained together securely using cryptography. There are also thousands of numbers and proportions that are used in it.

MATHEMATICS AND EDUCATION

FIRST STEPS

The first civilization which started learning and discovering mathematics was Ancient Greece. Here are some of the world-known Greek mathematicians: Pythagoras, Euclid, Thales of Miletus, Plato, etc.
Greeks at the beginning used mathematics only to measure, count something or for rituals, but by the 6th century B.C all the free Greeks (not including slaves) were educated and were studied, among others, mathematics. There were two types of schools: Palaestras (where pupils

only learned simple counting in mathematics lessons) and second type - Gymnasiums, private schools of higher level (where pupils were taught arithmetic and geometry).
Pythagoras school was saying:"Numbers rule the world" and "Nature speaks with us using the language of mathematics".
Euclid (325 B.C) who was regarded as "Father of geometry", wrote the famous book "Elements" based on thirteen essays and on works of Hippocrates, Leon and Feudius. In Elements there were basic geometrical axioms and theorems. Elements became the main book for learning math for the next two thousand years.

ANCIENT ROME

Roman math took many things from the Greeks. Roman numerals and Julian calendar are well-known in the world nowadays. The first mechanical calculator called abacus was invented in Ancient Rome. However, there weren't any famous mathematicians in Rome.
The education in Rome was different, rich families were sending their children to Greece or invited teachers to their houses. Whereat poor people attended ordinary schools that had three levels of education: primary, secondary and higher pupils learned counting only in primary school, then they were only taught theoretical subjects like history and law.

THE MEDIEVAL AGES

When the barbarians destroyed the Roman Empire, in the fifth century A.C. there came medieval period in Europe when there were terrible diseases and the majority of people weren't able to afford books and schools. Also many people were thinking about the church and their sins more than education. But then the first universities started to appear, e.g. in the eleventh century in the Italian city of Bologna. In Spain in the twelfth century scientists started to translate books and essays of Greeks and Arabians. Also in the twelfth century Paris University, Cambridge and Oxford were established.

EASTERN COUNTRIES

When the barbarians destroyed the Roman Empire, in the fifth century A.C. there came medieval period in Europe when there were terrible diseases and the majority of people weren't able to afford books and schools. Also many people were thinking about the church and their sins more than education. But then the first universities started to appear, e.g. in the eleventh century in the Italian city of Bologna. In Spain in the twelfth century scientists started to translate books and essays of Greeks and Arabians. Also in the twelfth century Paris University, Cambridge and Oxford were established.

RENAISSANCE TIME

After the medieval ages, at the end of the fifteenth century people started thinking about science and Byzantium became a knowledge centre of Europe. The sixteenth century was a turning point in mathematics, where a big breakthrough was made based on previous knowledge. Del Ferro Tartaglia and Ferrari came up with a method for solving cubic and quartic

equations,something that was considered impossible until then. In 1585 Simon Stevin published a book "Decimal" in which he explains actions with decimal fractions.
In the seventeenth century mathematics kept developing fast, but by the end of the century math has changed a lot. Rene Descartes in his book "Geometry" corrected the mistake of ancient scientists and restored algebraic understanding of numbers. Pierre Fermat, Jacob Bernoulli, Huygens created the theory of relativity.
At that time many schools and universities were built and more subjects related to mathematics were introduced at schools. The majority of people concentrated on learning math.

NOWADAYS

Nowadays modern math keeps developing, every day scientists find something new or improving mistakes of the past. Mathematical subjects like geometry and algebra are now in every school they are necessary to learn. In every exam we will see math and in order to get a place in a university we need to undertake a mother language exam and math exam.

MATHEMATICS AND MUSIC

Math and music have a lot in common – especially in frequencies. Let's talk about them.

HOW A STRING VIBRATES

When a string on a violin is vibrating, its ends are fixed to the instrument. This means the string can vibrate only on certain waves - sine waves (like a jump ropes), with different number of bumps between the ends. The more the bumps – the higher the pitch of a string is, and the faster the string is vibrating (we will get to that later.)
String (ends fixed to the instrument)
Sine waves with: 1 bump, 2 bumps

FREQUENCY OF A STRINS'S VIBRATION

The frequency of a string's vibration is the number of bumps multiplied by its fundamental frequency (frequency of a string vibrating with only 1 bump - when you play it without touching it).
"A" string's fundamental frequency is 440 Hz (cycles per second). We will use that information later.
Number of bumps
Frequency

1	F
2	2*F
3	3*F
4	4*F

"F" stands for string's fundamental frequency.

INTERVALS

Ratios between different frequencies have their own names. These are the intervals.
Octave – 2:1
Fifth – 3:2
Fourth – 4:3
Major Third – 5:4
Minor Third – 6:5
7:6 and 8:7 don't have names
Whole step – 9:8
Major Sixth – 5:3
Major Seventh – 15:8

DIATONIC SCALE

Diatonic scale is built using the base frequency and intervals.
It will be easier to show the diatonic scale of C than the diatonic scale of A, so I am going to find the frequency of C.
C D E F G A B
There are 6 steps (including C) between C and A, so we are going to use the ratio of Major sixth, 5:3. If C:A is 5:3, then A:C is 3:5. So, to get the frequency of C, we need to multiply A by 3 and divide by 5.
440 * 5:3 = 264
(Previously we said that the frequency of A is 440 Hz)
This is the diatonic scale of C Major. Everything seems to be all right until we change the key of the scale...
A lot of frequencies don't match. This is why musicians need to tune their instruments right if they want to change the key of the scale.
Equitemped (equal temped) scale was invited for a great reason:
It's impossible to tune a piano using diatonic methods! Each note has its own string, and there are 12 notes in each 7 octaves on an average piano. If we try to multiply perfect ratios from the diatonic scale, we'll never get 2 – we will never go an octave up perfectly.
So, equitempered scale is built on perfectly equal steps between these 12 semitones in an octave.
Let's try to find a formula to get the frequency of any note between A4 (440 Hz) and A5 (880 Hz, because we go an octave up).

A	A#	B	C	C#	D	D#	E	F	F#	G	G#	A
440	?	?	?	?	?	?	?	?	?	?	?	880
*x	*x	*x	*x	*x	*x	*x	*x	*x	*x	*x	*x	

$440 * x^{12} = 880$
$x^{12} = 2$
$x = \sqrt[12]{2} = \sim 1.0595$

Now we found our multiplier, so we can find all the missing frequencies.

A A# B C C# D D# E F F# G G# A

440|466.1|493.8|523.3|554.4|587.3|622.3|659.3|698.5|740.0|784.0|830.6|880
(*x) (* x) (* x) (* x) (* x) (* x) (* x) (* x) (* x) (* x) (* x) (* x)

x = 12√2
Any instrument tuned using equitempered scale can be used to play any key with a tiny differences in tune. As I said earlier, most of modern pianos are tuned to this scale.

MATHEMATICS AND SPORTS

At first glance, sports and math seemingly have little in common. However, a closer look at the sport reveals that there is a considerable amount of math in sports. Let's take basketball as an example.
The angle at which the ball is thrown is defined as the angle made by the extension of the player's arms and a perpendicular line starting from the player's hips.

GEOMETRY IN BASKETBALL

Whether they realize it or not, basketball players make use of many geometric concepts while playing a game. The most basic of these ideas is in the dimensions of the basketball court. The diameter of the hoop (18 in), the diameter of the ball (9.4 in), the width of the court (50 ft) and the length from the three point line to the hoop (19 ft) are all standard measures that must be adhered to in any basketball court.
The path the basketball will take once it's shot depends on the angle at which it is shot, the force applied and the height of the player's arms. When shooting from behind the free throw line, a small angle is necessary to get the ball through the hoop. However, when making a field throw, a larger angle is called for. When a defender is trying to block the shot, a higher shot is necessary. In this case, the elbows should be as close to the face as possible.
Understanding arcs will help determine how to best shoot the ball. Basketball players understand that throwing the ball right at the basket will not help it go into the hoop. On the other hand, shooting the ball in an arc will increase its chances of falling through the hoop. Getting the arc right is important to ensure that the ball does not fall in the wrong place.
The best height to dribble can also be determined mathematically. When standing in one place, dribble from a lower height to maintain better control of the ball. When running, dribbling from the height of your hips will allow you to move faster. To pass the ball while dribbling, use obtuse angles to pass the ball along a greater distance.
Understanding geometry is also important for good defence. This will help predict the player's moves, and also determine how to face the player. Facing the player directly will give the player

greater space to move on either side. However, facing the player at an angle will curb his freedom. Mathematics can also be used to decide how to stand while going on defence. The more you bend your knees, the quicker you can move. Utilizing geometry, math in basketball plays a crucial role in the actual playing of the sport.

STATISTICS IN BASKETBALL

Statistics essential for analysing a game of basketball. For players, statistics can be used to determine individual strengths and weaknesses. For spectators, statistics is used to determine the value of players and analyse the performance of an individual or the entire team. Percentages are a common way of comparing players' performances. They are is used to get values like the rebound rate, which is the percentage of missed shots a player rebounds while on the court. Statistics is also used to rank a player based on the number of shots, steals and assists made during a game. Averages are used to get values like the average points per game, and ratios are used to get values like the turnover to assist ratio.

MATHEMATICS AND TECHNOLOGIES

More than 35 years of active discussions involving academic representatives and practitioners: is math the Foundation of programming? Their opinions often vary greatly: some believe that information technology is derived from math, others argue that information technology is a separate science and math is just its equal partner. Here I will try to consider all points of view.
In the modern school, computers are increasingly used not only in science lessons but also in the lessons of mathematics, chemistry, biology, literature and fine arts. Information technologies not only facilitate access to information and the use of various educational activities, individualization and differentiation, but also allow new ways of interaction of all subjects of study and build an educational system in which student is an active and equal participant in educational activities.
The use of new information technologies allows you to replace many of the traditional means of learning. In many cases this substitution is effective, as it allows to maintain the students ' interest to the subject, creates an environment that stimulates the interest and curiosity of the student. At school computers allow the teacher to quickly combine a variety of tools that promote deeper and more conscious assimilation of the studied material, saves class time.
In mathematics lessons teachers use presentations that that they had created or found in the Internet, for the following reasons:

- To demonstrate to the students neat, clear samples of solutions
- To demonstrate concepts and objects;
- To achieve the optimal pace of students;
- To improve clarity in the course of training;
- To study more material;
- To show students the beauty of geometric drawings
- To increase cognitive interest;
- To introduce elements of entertainment to enliven the educational process;
- To achieve the effect of rapid feedback.

The intensity of mental activity during math class allows teachers to maintain students ' interest to the subject throughout the lesson
The use of information computer technologies for classes has led to an increase in interest in the subject of mathematics and experiment on their own thus becoming young researchers.
Mathematical methods are quite often used for data processing of information. They can act either as a constituent element of other methods, or independently. Such mathematical methods are used in Financial analysis, Decoding, Handwriting analysis. The role of mathematical methods in data processing increases significantly with the development of computers and information technology.

CONCLUSION

Despite the fact that mathematics appears grey and boring science, it is very diverse. It is difficult to find areas of human activities not related to mathematics. Our project touched only some areas, but there are many more, among them there are medicine, astronomy, theater and other. Since the Ancient times, mathematics has not only lost its former knowledge, it has been developing and improving human society.

LITERATURE AND RESOURCES

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MATHEMATICS IN COMPUTING

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ABSTRACT

Computing heavily relies on mathematics. Such algebras as elementary, abstract and Boolean are the most commonly used in coding.
Variables, equality, inequality and functions are concepts of elementary algebra applied in programming. The difference between mathematical and programming variables is that coding ones may be of different types. Programming equality and inequality do not differ from mathematical. Their application in coding is comparing variables. Functions in coding differ from mathematical ones a lot. In mathematics, a function is a relation between a set of inputs and a set of outputs with the property that each input is related to exactly one output, whereas in programming functions do not necessarily have an input and/or input.
Abstract algebra is taught during the first year of Bachelor in Computing due to its wide use in the subject. Groups, sets, modules, structures studied by abstract algebra, are used in computing. For instance, a checksum mechanism CRC relies on finite fields.
Boolean algebra is a form of mathematics that deals with statements and their Boolean values. It helps control the program flow depending on whether a programmer-specified Boolean condition evaluates to true or false. Languages with no explicit Boolean data type, like C90 and Lisp, may still represent truth values by some other data type.
Mathematics are also applied in scientific software. Mathematics together with computers assist in physics, chemistry, biology, geology and even mathematics themselves. Computers are also used to teach mathematics to children. Thousands of programs are designed to help children with mathematics.

• In elementary mathematics, a variable is an alphabetic character representing a number, called the value of the variable, which is either arbitrary, not fully specified, or unknown.
In computer programming, a variable or scalar is a storage location paired with an associated symbolic name (an identifier), which contains some known or unknown quantity of information referred to as a value.
In mathematics, equality is a relationship between two quantities or, more generally two mathematical expressions, asserting that the quantities have the same value, or that the expressions represent the same mathematical object.
In mathematics, an inequality is a relation that holds between two values when they are different
In computer science, a relational operator is a programming language construct or operator that tests or defines some kind of relation between two entities. These include numerical equality (e.g., 5 = 5) and inequalities (e.g., 4 ≥ 3).
In mathematics, a function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

In computer programming, a subroutine (also called a function) is a sequence of program instructions that perform a specific task, packaged as a unit.
• Abstract algebra (occasionally called modern algebra) is the study of algebraic structures, such as groups, rings, fields, modules, vector spaces, lattices, and algebras.
Group theory is very important in cryptography, for instance, especially finite groups in asymmetric encryption schemes such as RSA and El Gamal.
Another application of group theory, or, to be more specific, finite fields, is checksums. The widely-used checksum mechanism CRC is based on modular arithmetic in the polynomial ring of the finite field GF.
• Boolean algebra is a form of mathematics that deals with statements and their Boolean values. It is named after its inventor George Boole, who is thought to be one of the founders of computer science.
In computer science, the Boolean data type is a data type, having two values (usually denoted true and false), intended to represent the truth values of logic and Boolean algebra. Allows different actions and changes control flow depending on whether a programmer-specified Boolean condition evaluates to true or false.
Languages with no explicit Boolean data type, like C90 and Lisp, may still represent truth values by some other data type.
• Software that aids in research, testing or design. The software allows to keep complicated workflows based upon previous equations and chain those together to mock out a fully functioning system, where interconnected sensors would deliver various pieces of data to the overall equation.
• Computers are used in education in a number of ways, such as interactive tutorials, hypermedia, simulations and educational games. Tutorials are types of software that present information, check learning by question/answer method, judge responses, and provide feedback. Educational games are more like simulations and are used from the elementary to college level. The Incredible Machines is a good example of this type. E learning systems can deliver math lessons and exercises and manage homework assignments.

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THE MATHEMATICAL WAY OF WINNING AT MONOPOLY

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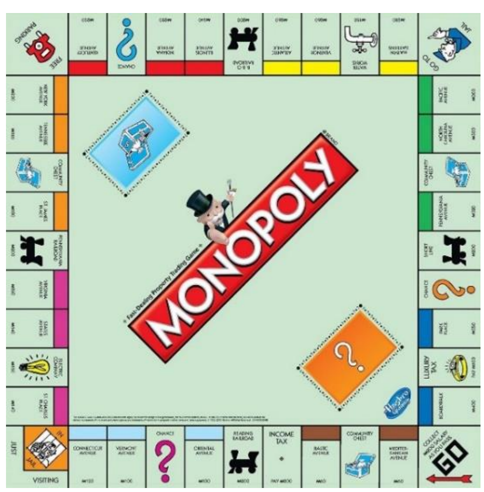
ABSTRACT

Everyone has played Monopoly at least once and either won excessively, controlling most of the properties and finances or lost everything in a horrible way. For the purposes of this paper, and for simplicity's sake, all wealth exchanges will be ignored. The interest lies only on the movement around the board, and the probability of ending a turn on a given field/property.

To understand the mathematics behind the answer to this problem, the game itself must be explained and understood first.

GAME MECHANICS

Monopoly is a board game played with 2 – 8 players. It is a game mimicking real life business and capitalism. All players start on the GO position and, throwing 2 six-sided dice, move around a 11 by 11 board. As they go they can buy almost all of the properties, which are all colour coded, excluding JAIL, GO TO JAIL, FREE PARKING, COMMUNITY CHEST fields… (Picture 1) The goal of the game is controlling most of the board's finances and make all other players go bankrupt. There are more aspects to the game too; like community chest and chance cards, the "go to jail" mechanic and many more.



Picture 1 – Monopoly board

LINEAR ALGEBRA AND MONOPOLY

All the mathematics of Monopoly can be subsumed under the Probability theory. Using Probability theory and linear algebra all movement probabilities can be calculated. It should be noted that, because linear algebra is a complex field of mathematics, all mathematics connected to it is simplified as well.
In this paper the following elements of linear algebra will be used: stochastic matrices, probability vectors and Markov chains.

SMALL SCALE EXAMPLE

Here's a small scale example. Consider a 2 by 2 board consisting of only the 4 corner pieces of the original Monopoly board (Picture 2). Also important is that only 1 die is used.

Picture 2 – 2 by 2 corner board

DICE MECHANICS

Assuming perfect randomness, all 6 die faces have a $\frac{1}{6}$ chance of appearing. Throwing 2 dice is different; there are more ways to get to the same value. One could, for example, get 3 in two different ways: throwing a 1 and a 2, or a 2 and a 1. This may not seem that different, but it does change things dramatically. (Picture 3)

Value	1	2	3	4	5	6	7	8	9	10	11	12
# of ways	0	1	2	3	4	5	6	5	4	3	2	1
Probability	0	1/36 (2.77%)	1/18 (5.55%)	1/12 (8.33%)	1/9 (11.11%)	5/36 (13.88%)	1/6 **(16.66%)**	5/36 (13.88%)	1/9 (11.11%)	1/12 (8.33%)	1/18 (5.55%)	1/36 (2.77%)

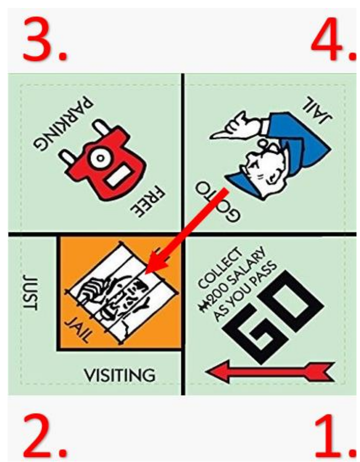
Picture 3 – Table of probabilities for throwing 2 dice

Following these rules, and taking the mechanic of the GO TO JAIL field into consideration, where you are moved to JAIL if you step on it, we can calculate the probability of ending the first turn, while starting on GO using stochastic matrices.

Stocastic matrices are algebraic opperators that consist only of probability vectors and essencialy map the probabilities of a set sistem at a set time.
Probability vestors are a mathematical function that describes a systems change of probability in a given time.
For this example the stocastic ma trix A is as follows: (Explanation, Picture 4 and 5)

$$A = \begin{array}{c} \\ 1. \\ 2. \\ 3. \\ 4. \end{array} \begin{array}{c} \begin{array}{cccc} 1. & 2. & 3. & 4. \end{array} \\ \begin{pmatrix} 1/6 & 1/6 & 1/3 & 0 \\ 1/2 & 1/2 & 1/2 & 1 \\ 1/3 & 1/3 & 1/6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Picture 4, Stocastic matirx for a 2 by 2 case



Picture 5, 2 by 2 corner grid, coresponding to numbering in Picture 4

The probabilities in the matrix correspond to the probabilities of a dice throw from the given position considering the rule associated to the 4th field.

MARKOV CHAINS

Markov chains - any mathematical function that can predict the probability of a set system only by its present states.
By definition it must satisfy the equation: $x_1 = Ax_0$, where x_1 is the state vector of the system after 2 dice throws, A is the stochastic matrix associated with the system, and x_0 is the first probability vector of the system (Picture 4, column 1).
The equation can be generalized to: $x_{n+1} = Ax_n$.
In the limit, as n tends to infinity, the probability vector, x_n converges to a set value, a steady-state vector. (Picture 6)

A steady-state vector is a vector which represents the converged values of a Markov chains, in other words, the probability of landing on a given square at a given time.

$$\overset{x_1}{\begin{pmatrix} 2/9 \\ 1/2 \\ 5/18 \\ 0 \end{pmatrix}} = \overset{A}{\begin{pmatrix} 1/6 & 1/6 & 1/3 & 0 \\ 1/2 & 1/2 & 1/2 & 1 \\ 1/3 & 1/3 & 1/6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}} \cdot \overset{x_0}{\begin{pmatrix} 1/6 \\ 1/2 \\ 1/3 \\ 0 \end{pmatrix}} \Longrightarrow \overset{x_2}{\begin{pmatrix} 23/108 \\ 1/2 \\ 31/108 \\ 0 \end{pmatrix}} = \overset{A}{\begin{pmatrix} 1/6 & 1/6 & 1/3 & 0 \\ 1/2 & 1/2 & 1/2 & 1 \\ 1/3 & 1/3 & 1/6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}} \cdot \overset{x_1}{\begin{pmatrix} 2/9 \\ 1/2 \\ 5/18 \\ 0 \end{pmatrix}}$$

Picture 6, Extended Markov

In this case it converges to $x_n = \begin{pmatrix} 0.21429\ldots \\ 0.5 \\ 0.28571\ldots \\ 0 \end{pmatrix}$, therefore the probability solution for a 2 by 2 board is represented in Picture 7.

Field	**1.**	**2.**	**3.**	**4.**
Turn 1	1/6 (16.66%)	½ (50%)	1/3 (33.33%)	0 (0%)
Turn n	**21%**	**50%**	**29%**	**0%**

Picture 7, Probability table for a 2 by 2

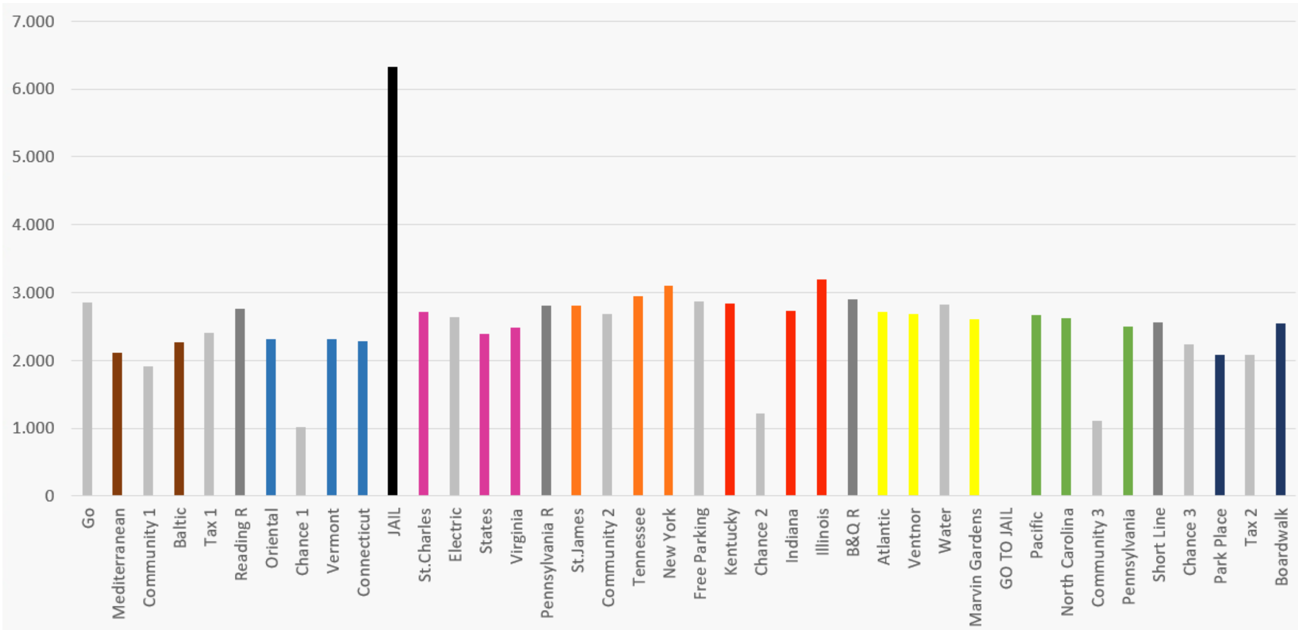
SCALED UP CALCULATION

Applying the same rules and calculations to a 11 by 11 board.

$$x_1 = Ax_0 \Longrightarrow x_2 = Ax_1 \Longrightarrow x_3 = Ax_2 \Longrightarrow \ \ldots \ \Longrightarrow x_n = Ax_{n-1}$$

Picture 8, Extended Markov chain (general)

Using the Markov chain extension (Picture 8) and a stochastic matrix A with 21 rows and columns, the following results are obtained (Picture 9 and Picture 10):



Picture 9, Graph of the final probabilities

Property Name	Probability	Property Name	Probability
Go	2.854%	Free Parking	2.868%
Mediterranean Avenue	2.109%	Kentucky Avenue	2.839%
Community Chest 1	1.919%	Chance 2	1.212%
Baltic Avenue	2.261%	Indiana Avenue	2.731%
Income Tax	2.404%	Illinois Avenue	3.197%
Reading Railroad	2.757%	B&O Railroad	2.897%
Oriental Avenue	2.317%	Atlantic Avenue	2.719%
Chance 1	1.017%	Ventnor Avenue	2.689%
Vermont Avenue	2.308%	Water Works	2.821%
Connecticut Avenue	2.278%	Marvin Gardens	2.601%
JAIL	6.325%	GO TO JAIL	0%
St. Charles Place	2.719%	Pacific Avenue	2.675%
Electric Company	2.61%	North Carolina Avenue	2.616%
States Avenue	2.395%	Community Chest 3	1.116%
Virginia Avenue	2.477%	Pennsylvania Avenue	2.494%
Pennsylvania Railroad	2.804%	Short Line	2.553%
St. James Place	2.8%	Chance 3	2.239%
Community Chest 2	2.62%	Park Place	2.085%
Tennessee Avenue	2.94%	Luxury Tax	2.086%
New York Avenue	3.096%	Boardwalk	2.552%

Picture 10, Table of the final probabilities

SUMMARY

- Jail is the most visited field.
- Jail increases the probability of fields coming after it.
- Orange and red properties are the most valuable by this theory.
- Dark blue and brown properties are the least valuable.
- It follows from the research and wealth calculation that it is better to buy 4 houses then a hotel – “The monopoly on houses”.

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CALCULATING CUBE ROOT

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ABSTRACT

The cube root of a number means it is a value of that, when used in a multiplication by itself in three times, gives that number. In mathematics, a cube root of a number x is a number y such that y3 = x. In this paper explained about one simple and easy tip for finding Cube Roots of Perfect Cubes of two digits numbers. By this cube root formula we find cube root in fraction of seconds. These points to be remember for this cube root formula. The given number should be perfect cube. Remember cubes of 1 to 10 numbers. For per cubes identify as follow as Cube table 1 to 9. Identify the last three digits from right side and make group of these three digits. Take the last group and then find the cube root of it. Hence the right most digit of the cube root of the given number is obtained. Take the next group. Find out the value of it. The given number lies in between the cube of the numbers a^3 and b^3. Take small cube number. Hence the left neighbor digit of the answer is obtained. So our answer is solved and cube root is obtained. In this paper two methods were proposed to calculate the cube of numbers. One of these methods is using the power table 3 for number 0 to 9 and the cube of numbers, and the other is the decomposition of the number into the first factors and the extraction of the numbers with the power of 3.

FINDING CUBE ROOT OF A NUMBER QUICKLY

The third root is a radical number with a subfield 3 that is the third represented by the symbol $\sqrt[3]{x}$. If the number below the radical with the 3rd order reaches 3, the power of the number with the simple power is simplified and the number is released from the sub-radical. To calculate the third root of 64, it can be written as $\sqrt[3]{4^3}$ with its third root equal to 4, and to calculate the third root of 64, it can be written as $\sqrt[3]{5^3}$, that the third root It is equal to 5. In this paper, different modes are calculated for calculating the third root, and the simplest method for each mode is proposed And also in this paper explained about one simple and easy tip for finding Cube Roots of Perfect Cubes of two digits numbers. By this cube root formula we find cube root in fraction of seconds.

THE STEPS TO FIND THE THIRD ROOT

Keep in mind the cubes of 0 to 9

Number	Cube	Last Digit
0	0	0
1	1	1
2	8	8
3	27	7
4	64	4
5	125	5
6	216	6
7	343	3
8	512	2
9	729	9

Note how the last digit of the cubes for 0, 1, 4, 5, 6, and 9 end with the original number. Note how the last digits for the cubes of 2 and 8 are swapped. Note how the last digits for the cubes of 3 and 7 are swapped. To master the system you must learn by heart the cubes of numbers 0 to 9, which are shown in the table below. You also need to consider the last digit of each cube. These points to be remember for this cube root formula.
1. The given number should be perfect two digit cube. 2. Remember cubes of 1 to 10 numbers.
3. As per the cubes identify as follow as below table.

	If last digit of perfect cube number =	last digit of cube root for that number=
13 = 1	1	1
23 = 8	8	2
33 = 27	7	3
43 = 64	4	4
53 = 125	5	5
63 = 216	6	4
73 = 343	3	7
83 =512	8	2
93 = 729	9	9
103 = 1000	0	0

DETERMINE THE CUBE ROOT

Ignore the last three digits of the number called out and choose the memorized cube which is just lower (or equal) to the remaining number. The cube root of this is the first digit of your answer. Now consider the last digit of the number called out. This will indicate the last digit of your answer. For example, if the last digit of the number called out is 3, then the last digit of the cube root is 7 (see the last digit values in the table above).
Examples:

Called Numbers	Ignore last 3	Lower Cube	First Digit	Last Digit	Cube Root
1728	1	1	1	2	12
21952	21	8	2	8	28
50653	50	27	3	7	37
117649	117	64	4	9	49
148877	148	125	5	3	53
216000	216	216	6	0	60
357911	357	343	7	1	71
636056	636	512	8	6	86
830584	830	729	9	4	94

Now let’s see how we can easily find out cube roots of perfect cubes with in fraction of seconds. Take examples to easily understand the cube root formula.

Example 1: We get the third root of 405224 as follows:

$$\sqrt[3]{405224} = \mathrm{cd}$$

0	0	0
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9

Step 1: Identify the last three digits from right side and make group of these three digits: i.e., 405 – 224 ; A=405 ; B=224

Step 2: Take the last group which is 224. The last digit of 224 is 4. According to above table if last digit having 4 then last digit of cube root for that number is 4. Hence the right most digit of the cube root of the given number is 4. So we have: d=4.

Step 3: Take the next group which is 405. Find out the value of 405 lies in between the cube of the numbers 73 and 83. Then 343 < 405 < 512. Take small cube number i.e. "7". Hence the left neighbor digit of the answer is 7. So our answer = 74.

$$\sqrt[3]{405224} = 74$$

Example 2: We get the third root of 97336 as follows:

$$\sqrt[3]{97336} = \mathrm{cd}$$

0	0	0
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9

Step 1: Identify the last three digits from right side and make group of these three digits: i.e., 97 – 336; A=97; B=336.

Step 2: Take the last group which is 336. The last digit of 336 is 6. According to above table if last digit having 6 then last digit of cube root for that number is 6. Hence the right most digit of the cube root of the given number is 6. So we have: d=6.

Step 3: Take the next group which is 97. Find out the value of 97 lies in between the cube of the numbers 64 and 125. Then 64 < 97 < 125. Take small cube number i.e. "4". Hence the left neighbor digit of the answer is 4. So our answer = 46.

$$\sqrt[3]{97336} = 46$$

Example 3: We get the third root of 274625 as follows:

$$\sqrt[3]{274625} = \mathrm{cd}$$

0	0	0
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9

Step 1: Identify the last three digits from right side and make group of these three digits: i.e., 274 – 625; A=274; B=625.

Step 2: Take the last group which is 625. The last digit of 625 is 5. According to above table if last digit having 5 then last digit of cube root for that number is 5. Hence the right most digit of the cube root of the given number is 5. So we have: d=5

Step 3: Take the next group which is 274. Find out the value of 274 lies in between the cube of the numbers 216 and 343. Then 216 < 274 < 343. Take small cube number i.e. “6 “. Hence the left neighbor digit of the answer is 6. So our answer = 65.

$$\sqrt[3]{274625} = 65$$

Example 4: We get the third root of 857375 as follows:

$$\sqrt[3]{857375} = \mathrm{cd}$$

0	0	0
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9
10	1000	0

Step 1: Identify the last three digits from right side and make group of these three digits: i.e., 857-375; A=857; B=375.

Step 2: Take the last group which is 375. The last digit of 375 is 5. According to above table if last digit having 5 then last digit of cube root for that number is 5. Hence the right most digit of the cube root of the given number is 5. So we have: d=5.

Step 3: Take the next group which is 857. Find out the value of 857 lies in between the cube of the numbers 729 and 1000. Then 729 < 857 < 1000. Take small cube number i.e. "9 ". Hence the left neighbor digit of the answer is 9. So our answer = 95.

$$\sqrt[3]{857375} = 95$$

Example 5: We get the third root of 54872 as follows:

$$\sqrt[3]{54872} = \mathrm{cd}$$

0	0	0
1	1	1
2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8

Step 1: Identify the last three digits from right side and make group of these three digits: i.e., 54 – 872; A=54; B=872.

Step 2: Take the last group which is 872. The last digit of 872 is 2. According to above table if last digit having 2 then last digit of cube root for that number is 8. Hence the right most digit of the cube root of the given number is 5. So we have: d=8.

Step 3: Take the next group which is 54. Find out the value of 54 lies in between the cube of the numbers 729 and 1000. Then 27 < 54 < 64. Take small cube number i.e. "3 ". Hence the left neighbor digit of the answer is 3. So our answer = 38.

$$\sqrt[3]{54872} = 38$$

FINDING THE THIRD ROOT BY PRIME FACTORIZATION

We can find the cube root of a number by the method of prime factorization. Consider the following example for a clear understanding: 2744 = 2 × 2 × 2 × 7 × 7 × 7 = $(2 \times 7)^3$. Therefore the cube root of 2744 = $\sqrt[3]{2744} = 2 \times 7 = 14$.
The steps to find the third root of the numbers are: Step 1. Decomposing the numbers into the first counters. Step 2. Sort the counters with power. Step 3. Find the number that has the power of 3. Step 4. The number that powers 3 can be released with a simplified substrate from the radical. If the group of decomposable numbers has no power 3, then there is no third digit and the cube is not complete. The following examples help to solve examples for the third root.

Example 1: Find the third root of the number 216 as follows:

We divide the number into the Prime Factorization

216	2
108	2
54	2
27	3
9	3
3	3
1	

So the number 216 can be written as follows:

216 = 2 × 2 × 2 × 3 × 3 × 3 = (2 × 2 × 2) × (3 × 3 × 3)
$\sqrt[3]{216} = \sqrt[3]{(2 \times 3)^3} = \sqrt[3]{6^3} = 6$

Example 2: Find the third root of the number 343 as follows:

We divide the number into the Prime Factorization:

343	7
49	7
7	7
1	

So the number 216 can be written as follows:
343 = 7 × 7 × 7 = (7 × 7 × 7) → $\sqrt[3]{343} = \sqrt[3]{7^3} = 3$

Example 3: Find $\sqrt[3]{46656}$ by the method of prime factorization.
Let us first find the prime factors:

46656	2
23328	2
11664	2
5832	2
2916	2
1458	2
729	3
243	3
81	3
27	3
9	3
3	3
1	

46656 = 2×2×2×2×2×2×3×3×3×3×3×3 = 23×23×33×33 = $(2\times2\times3\times3)^3$
Therefore, $\sqrt[3]{46656} = 36$

Example 4: Find $\sqrt[3]{2744}$ by the method of prime factorization.
Let us first find the prime factors:

2744	2
1372	2
686	2
343	7
49	7
7	7
1	

2744 = 2 × 2 × 2 × 7 × 7 × 7= (2 × 2 × 2) × (7 × 7 × 7) →
$\sqrt[3]{2744}$ = (2 × 7) = 14

IDENTIFY COMPLETE CUBE NUMBERS

The number is broken down by the column or ladder method to the prime factors. If all the prime factors are multiplied by 3 times, then the number is complete cube.

Example 1: Consider the number 250:
Parsing it into the prime factors is as follows:

250	2
125	5
25	5
5	5
1	

250 = 2 × 5 × 5 × 5

The factor 2 is power 1 and does not have power 3. Therefore, the number 250 is not complete cube.

Example 2: Consider the number 5832. Parsing it into the prime factors is as follows:

5832	2
2916	2
1458	2
729	3
243	3
81	3
27	3
9	3
3	3
1	

5832 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

All factors have the power of 3. So the number 5832 is complete cube.

Example 3: Consider the number 5832. Parsing it into the prime factors is as follows:

1994	2
972	2
486	2
243	3
81	3
27	3
9	3
3	3
1	

1944 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3

All factors have the power of 3. So the number 5832 is complete cube. After breaking into the prime factors, it is seen that one of the 3s has a power of 3 and another 3 has a powers of 2. So the number 1994 is not complete cube. This number must be multiplied by 3 to complete the cube.

CONCLUSION

Two methods were proposed to calculate the cube of numbers. One of these methods is using the power table 3 and the cube of numbers, and the other is the decomposition of the number into the first factors and the extraction of the numbers with the power of 3.

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THE ROLE OF MATHEMATICS IN MEDICINE

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ABSTRACT

In this article, you will find out many important things about the role of mathematics in medicine. For example, you will find out how doctors use mathematics to write prescriptions; You also discover the relationship between ellipse and treating gallstones and kidney stones; you see medical graphs too. You also learn that mathematics play an important role in the treatment of cancer and tumors. Finally, you will become aware of the most interesting role of mathematics, a model that predicts outbreaks of infectious diseases.

INTRODUCTION

Today, at advanced levels of research, mathematics has played a very important role in science and medical studies. So that mathematical equations cannot be denied in different parts of medicine. Undoubtedly, close interactions between physicians and mathematicians will contribute to a more principled and more effective treatment, and the physician can use the mathematical equations in the treatment of diseases and by using clinical and disease management methods to fundamentally treat illnesses, to reduce the Side effects of medications in treatment.
In Medicine three subjects are specially considered:

1. Diagnose the disease
2. Cure the disease
3. Control the disease

Infectious and non-infectious disease are a dynamic system. So, they have minor and ordinary Differential Equations. They have such equations for Diabetes, HIV, Tuberculosis, Hepatitis, Tumor… That they have been determined now and these mathematical models are getting progress. In those three subjects mathematics have an incredible role that its result can help medicine a lot.

WRITING PRESCRIPTIONS

Regularly, doctors write prescriptions to their patients for various ailments. Prescriptions indicate a specific medication and dosage amount. Most medications have guidelines for dosage amounts in milligrams (mg) per kilogram (kg). Doctors need to figure out how many milligrams of medication each patient will need, depending on their weight. If the weight of a patient is only known in pounds, doctors need to convert that measurement to kilograms and then find the amount of milligrams for the prescription. There is a very big difference between mg/kg and mg/lbs, so it is imperative that doctors understand how to accurately convert weight measurements. Doctors must also determine how long a prescription will last. For example, if a patient needs to take their medication, say one pill, three times a day. Then one month of pills is approximately 90 pills. However, most patients prefer two or three month prescriptions for convenience and insurance purposes. Doctors must be able to do these calculations mentally with speed and accuracy.

Doctors must also consider how long the medicine will stay in the patient's body. This will determine how often the patient needs to take their medication in order to keep a sufficient amount of the medicine in the body. For example, a patient takes a pill in the morning that has 50mg of a particular medicine. When the patient wakes up the next day, their body has washed out 40% of the medication. This means that 20mg have been washed out and only 30mg remain in the body. The patient continues to take their 50mg pill each morning. This means that on the morning of day two, the patient has the 30mg left over from day one, as well as another 50mg from the morning of day two, which is a total of 80mg. As this continues, doctors must determine how often a patient needs to take their medication, and for how long, in order to keep enough medicine in the patient's body to work effectively, but without overdosing.

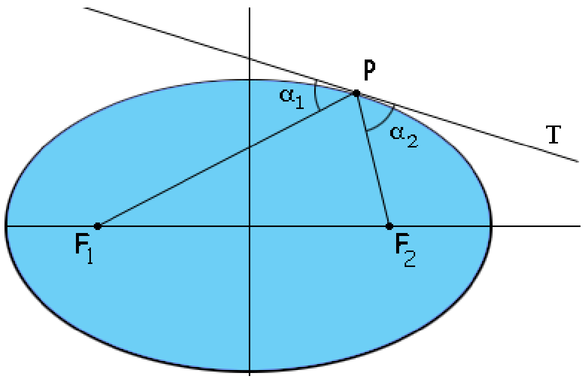
The amount of medicine in the body after taking a medication decreases by a certain percentage in a certain time (perhaps 10% each hour, for example). This percentage decrease can be expressed as a rational number, 1/10. Hence in each hour, if the amount at the end of the hour decreases by 1/10 then the amount remaining is 9/10 of the amount at the beginning of the hour. This constant rational decrease creates a geometric sequence. So, if a patient takes a pill that has 200mg of a certain drug, the decrease of medication in their body each hour can be seen in the folowing table. The Start column contains the number of mg of the drug remaining in the system at the start of the hour and the End column contains the number of mg of the drug remaining in the system at the end of the hour.

The sequence of numbers shown below is geometric because there is a common ratio between terms, in this case 9/10. Doctors can use this idea to quickly decide how often a patient needs to take their prescribed medication.

Hour	Start	End
1	200	9/10 x 200 = 180
2	180	9/10 x 180 = 162
3	162	9/10 x 162 = 145.8
.	.	.

ELLIPSE, KIDNEY STONES & GALLSTONES

The ellipse is a very special and practical conic section. One important property of the ellipse is its reflective property. If you think of an ellipse as being made from a reflective material then a light ray emitted from one focus will reflect off the ellipse and pass through the second focus. This is also true not only for light rays, but also for other forms of energy, including shockwaves. Shockwaves generated at one focus will reflect off the ellipse and pass through the second focus. This characteristic, unique to the ellipse, has inspired a useful medical application. Medical specialists have used the ellipse to create a device that effectively treats kidney stones and gallstones. A lithotripter uses shockwaves to successfully shatter a painful kidney stone (or gallstone) into tiny pieces that can be easily passed by the body. This process is known as lithotripsy.

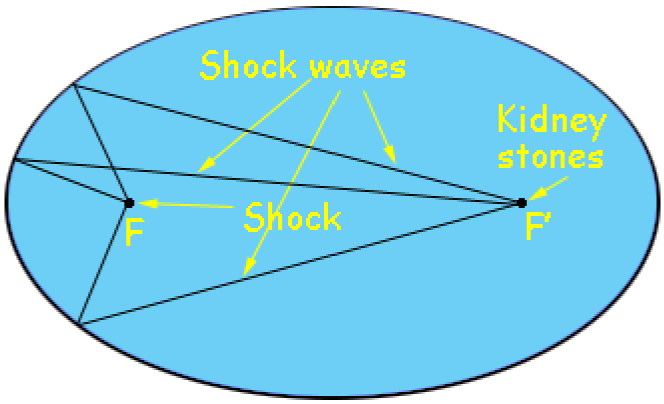


As illustrated in the diagram above, when as energy ray reflects off a surface, the angle of incidence is equal to the angle of reflection.

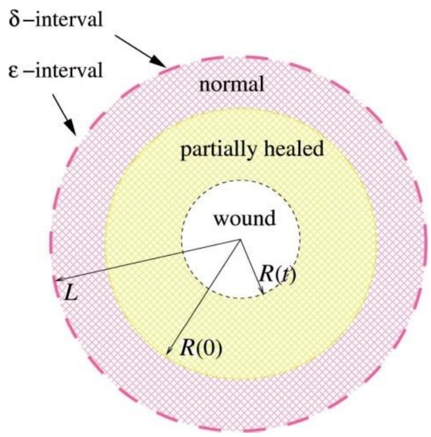
$$\alpha_1 = \alpha_2$$

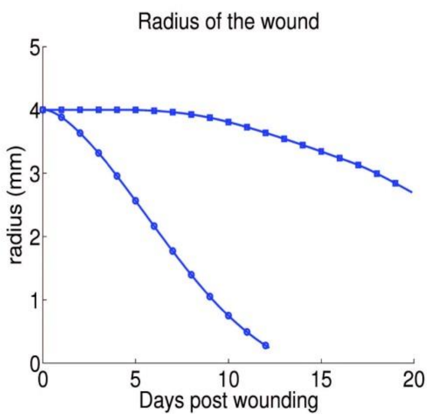
Extracorporeal Shockwave Lithotripsy (ESWL) enables doctors to treat kidney and gall stones without open surgery. By using this alternative, risks associated with surgery are significantly reduced. There is a smaller possibility of infections and less recovery time is required than for a surgical procedure. The lithotripter is the instrument used in lithotripsy. The mathematical properties of an ellipse provide the basis for this medical invention.

The lithotripter machine has a half ellipsoid shaped piece that rests opening to the patient's side. An ellipsoid is a three dimensional representation of an ellipse. In order for the lithotripter to work using the reflective property of the ellipse, the patient's stone must be at one focus point of the ellipsoid and the shockwave generator at the other focus. The patient is laid on the table and moved into position next to the lithotripter. Doctors use a fluoroscopic x-ray system to maintain a visual of the stone. This allows for accurate positioning of the stone as a focus. Because the stone is acting as one of the focus points, it is imperative that the stone be at precisely the right distance from the focus located on the lithotripter. This is essential in order for the shockwaves to be directed onto the stone.

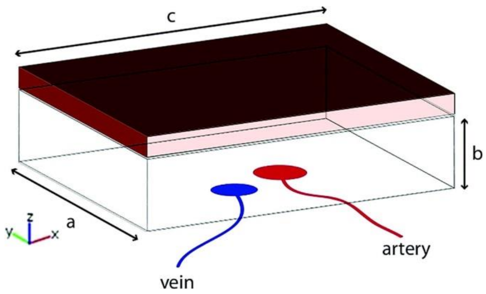


MEDICAL GRAPHS

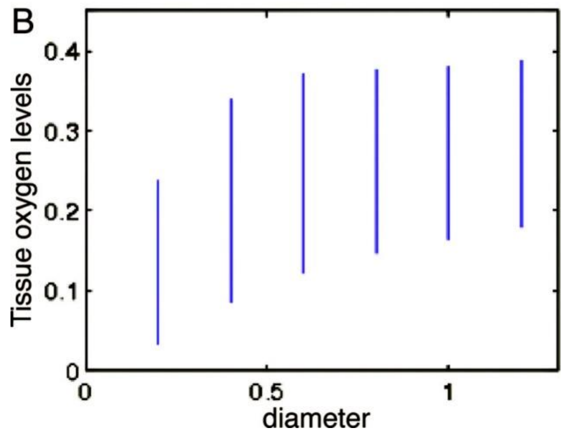




The open wound is the circular region {0 ≤ r ≤ R(t)}, the partially healed region is the annulus {R(t) ≤ r ≤ R(0)}, and the normal healthy tissue is {R(0) ≤ r ≤ L}.



Schematic of tissue flap. The top colored layer consists of dermis, epidermis,and subdermal plexus. The bottom layer represents fat tissue. The perforator artery and vein are located at the bottom of the flap.



Range of oxygen concentration in the flap of dimension 4 cm x 2.5 cm x 1 cm with different arterial diameter ranging 0.2 cm to 1.2 cm.

CURING TUMOR & CANCER WITH THE HELP OF MATHEMATIC

One of the principal methods of research in medical sciences is the use of mathematics because disease growth has laws that are now sought to be discovered. Mathematical equations are used in various medical departments, including treatment methods for the growth of tumors.
The coping model shows how the temperature of the tumor can be reached to 45 ° without affecting the surrounding tissue. If the heat is applied to the tissue of the tumor through the skin, healthy tissues will destroy it.
Through mathematical models, it is possible to determine how much time it takes to eliminate the damage and prevent damage to the surrounding tissues due to the growth and replication of the viruses as well as the range of the target tumor.
By using mathematical science, diseases are modeled and the diagnosis and treatment process is done at time and at a lower cost.

A group of American scientists have developed a computer model that can provide a combination of the most effective therapies for treating cancer using math algorithms.
From the combination of oncology and mathematics, you can get the greatest chance of identifying and recognizing effective treatments with the tumors.
Unfortunately, there's no available information about these useful models.

A MODEL TO PREDICT THE OUTBREAK OF THE INFECTIOUS DISEASES

American scientists have developed math algorithms that can help predict epidemics associated with the most common infectious diseases based on weather parameters.
Researchers at the Tufts University School of Medicine in Boston have presented a mathematical model that assesses the probability of an outbreak of these diseases on a daily basis, based on the environmental parameters on each site, according to Health News.
Scientists have tested their math model based on data collected by the Massachusetts Public Health Department regarding six diseases, according to the Medical News Todd.
The six diseases were Jardia and Cryptosporidium (two intestinal infectious diseases), Salmonella and Campylobacter (two common intestinal diseases that occur in the intestines due to the introduction of Salmonella and Campylobacter bacteria and are very common in Europe), Shigellosis (A tropical disease that occurs as a result of infection with Shigella bacteria) and A-hepatitis A due to infection with the Hiv virus.
Then, using these climatic data collected between 1992 and 2001, the scientists examined the incidence of each of these diseases in Massachusetts based on the values of the average daily temperature, the time and the course of each of these diseases.
The preliminary results of the experiment showed that the incidence of these diseases is related to hepatocellular carcinoma A, other than heat.
Therefore, the current algorithmic models are based on seasonal and monthly information on the epidemiology of infectious diseases, while the new model is being investigated daily.

CONCLUSION

Mathematics plays a crucial role in medicine and because people's lives are involved, it is very important for nurses and doctors to be very accurate in their mathematical calculations. Numbers provide information for doctors, nurses, and even patients. Numbers are a way of communicating information, which is very important in the medical field
Doctors can have a better diagnosis, curing and controlling with the help of mathematics.

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FRACTAL STRUCTURE OF DNA

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ABSTRACT

Fractals are formed from self-similar objects that ensue by self-repeating pattern thus forming smaller look-alike parts same as the entire object. Although Mandelbrot is considered to be the father of fractals, they owe its true existence to Giuseppe Peano, who defined a group of self-similar curves by which these are explained. Micro and macro world are both of fractal nature and so is, by definition, DNA, being a part of micro world itself. The paper will explain how fractal model of DNA, thanks to its characteristics, represents the most suitable way for DNA to fold, while comparing it to less suitable polymer model. Ever since the idea was first unveiled to scientific public, it has had a great influence on DNA structure analysis. Fractals started being widely used in various fields.

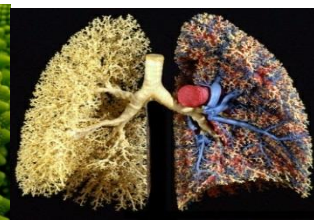
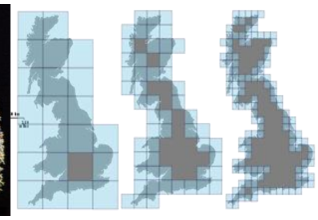
1. INTRODUCTION

The term ‘fractal’ first appeared in Ancient Rome as fractus, which means broken. Gottfried Leibniz, German mathematician and philosopher, first used the collocation of it to define and explain self-similarity of a straight line. Helge von Koch, Waclaw Sierpinski, Felix Hausdorf and Paul Levi made a great endowment to this mathematical discipline and Benoit Mandelbrot unified all then-known characteristics of fractals referring to them as to geometric structures whose fractal dimension is bigger than their topological one. Still, the simplest description says that a fractal is a mathematical assemblage of self-similar objects that ensue by repeating one identical procedure.

The world is full of fractals, but can we apply them on things we cannot perceive or even on ourselves? Absolutely. Universe is a fractal so as many organs in our body: our brain, lungs, blood vessels, whole nervous system. All systems of organs are of tissues generated by cells and these are made of cytosol and organelles. The most perplexed one, nucleus, keeps in its inwardness deoxyribonucleic acid (DNA), a genetic material which is the key to functioning of all organisms. In this scientific work we will explain how meters of DNA are packed within the nucleus’s radius of 5μm.

2. FRACTALS IN NATURE

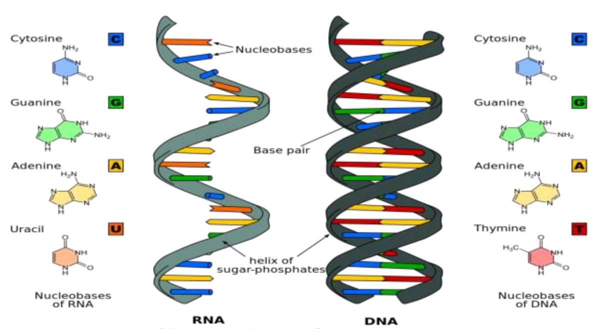
Broccoli, snowflakes, bacteria, fungi, trees, lightening, river flows and steep mountain slopes are just some of many fractal examples. As a part of the Universe we are also fractals. The most complex structure in the whole known Universe is a fractal, and it is within us- the Brain. Besides being rich with vitamins K and D, β-carotene and dietary fiber, Broccolo Romanesco of Brassica Oleracea species is a fascinating illustration of a fractal. Every single bud acts as a duplicate of the previous one thus building a seemingly endless logarithmic spiral. All coast lines ensued from geological processes are fractals which length can be measured using the fractal, Box-counting dimension that shows how much space the fractal fulfills. This method is explained in Mandelbrot's book "How long is the coast of Britain?". Another proof of fractal geometry in nature gave Michael Barnsley in his book "Fractals everywhere", where he defines formulas for counting dimensions of the fern's structure.

Picture n. 1 Broccolo Romanesco   Picture n. 2 Lungs
Picture n. 3 Coast of Great Britain measured by Box-counting dimension

3. CHARACTERISTICS AND STRUCTURE OF DNA
3.1 BASICS OF DNA

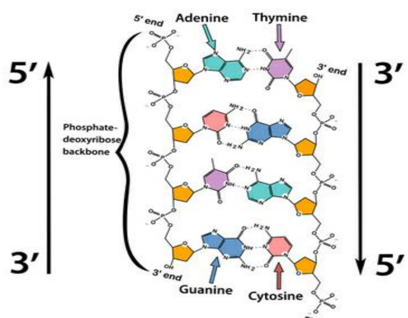
Nucleic acids are constructed of three components: phosphate groups, carbohydrates- pentosis and nitric bases. The hydrates type, ribose or deoxyribose, is the main divider for ribonucleic and deoxyribonucleic acids. Nitric bases are classified by pirins, and those are adenine and guanine, and pyrimidines, thymine, cytocine and uracil. The identical disposition of base pairs (bps), defined by one pirine and one pyrimidine does not exist in any two molecules of DNA and that is the specificity of DNA chains. Their nucleotides are coded by genes. Three basic functions of DNA are transcription, translation and replication.



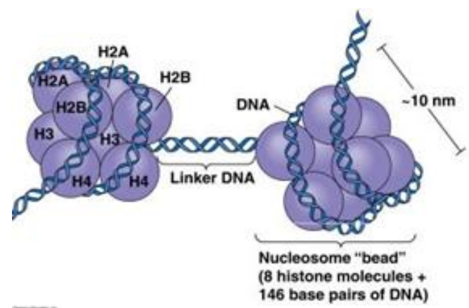
Picture n. 4 DNA and RNA and their nitric bases

3.2 STRUCTURE OF DNA

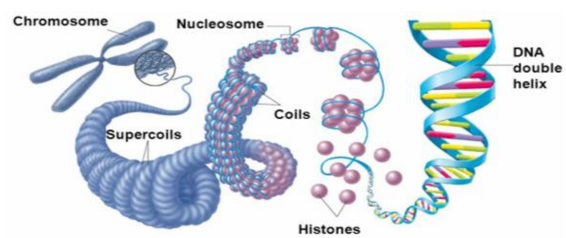
DNA molecule is formed from two spirally twisted polynucleotide chains connected by hydrogen bonds between nucleobases. Carbohydrates and phosphate groups connected with 3', 5' phosphodiestar bonds make the backbone of helixes, while overside are nucleobases. Two polynucleotide threads are directed in the opposite directions so one goes from 5' to 3' and the other one from 3' to 5'. Deoxyribonucleic acid is a genetic material located in the nucleus, in fact in chromatin which is a hierarchical structure made from 150 bps and in the period of cell division it forms chromosomes. Each chromosome carries one DNA chain that folds multiple times on more than a few levels. The main and the most obvious reason for this modus of packing is the length of the chromatin formation. With the length between 1.8 and 2 meters, unpacked human DNA would surpass every second person on Earth. According to the latest research the number of cells in our body is 37.2 trillion so knowing that every human cell has 23 chromosomes and every chromosome one DNA chain, we can calculate the length of the total genetic material from our organism getting $1.54008*10^{12}$ kilometers, which is, in fact, ten thousand times longer than the distance between Sun and Earth! Surely, DNA also exists outside the nucleus (satellite DNA) but in, we can freely say, negligible amounts. The basic building unit of chromatin is a nucleosome made of histones and deoxyribonucleic acid. The base for this structure is a cubic octamer formed from eight nucleosomal histones, out of which every two are same. The DNA chain is wrapped around the octamer 1.6 times and sideward is the ninth, linear histone. All together, they make out deoxyribonucleic solenoid structure.



Picture n. 5 Phosphodiestar bond



Picture n. 6 Nucleosome



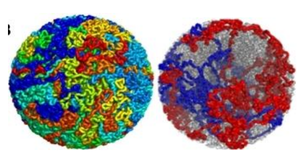
Picture n. 7 Stages of DNA packing

4. DNA FOLDING

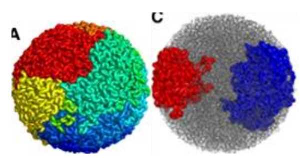
Polymer in which monomers that are at a great distance in the chain don't react with each other is called an ideal chain. Under specific circumstances the behavior of a polymer can be accurately defined by using the ideal chain. DNA can be packed in two ways, equilibrium or fractal. In the following text we will be explaining the properties and structure of these globules, as well as which one is more convenient and why.

4.1 EQUILIBRIUM GLOBULE

Equilibrium globule (EG) is a disorganized DNA structure that appears when the force of attraction between monomers overpowers the force of repulsion or when the monomers are compressed into a small enough volume thus changing the spiral form. The size of EG increases with the upsurge of the polymers length with the relation: R~N1/3, where R is the polymer's characteristic size and N the length of the one. Here, chromatin is, in the interphase of cell division, unregulated and chaotic packed with very distant parts of the chain compressed and placed close together. This unregulated form is the main property that describes the equilibrium state. With this, the "neighboring" DNA parts, which would have been assembled very close in a fractal globule (FG), are very distant. Tangling into the EG requires a lot of time considering the very slow motion of the chain and the need to make countless loops to decrease its volume as well.



Picture n. 8 Equilibrium globule



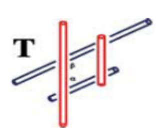
Picture n. 9 Fractal globule

4.2 FRACTAL GLOBULE

Fractal compact polymer state of DNA occurs during the condensation of certain polymer molecules as a result of preventing some parts of the chain to cross onto the territory of other. Since it fulfills the space of the nucleus entirely, its volume increases linearly with the upsurge of the polymers length, the same as the EG.

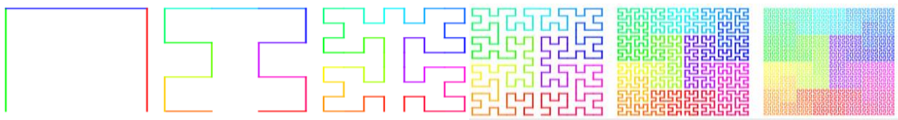
5. FRACTAL FOLDING

The polymer nature serves as a perfect example of fractal organization. DNA folds over its separated helixes by two molecules thus giving a panel like structure formed of four separated helixes, out of which every two are same and always parallel. Picture n.10 shows how in every helix pair one is shorter than the other. By arranging and binding these primary formations, we obtain more complex, chained structures. Predispositions for these are specific codes and matching of their geometric boundaries.



Picture n. 10 Panel like structure of DNA

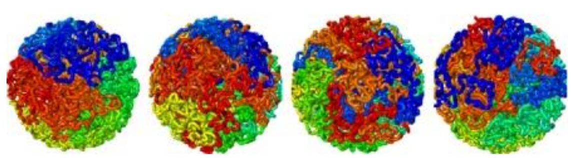
DNA, conformed into a FG, represents an endlessly dense curve called Peano curve. This curve and every other discovered after it got its name by Giuseppe Peano, an Italian mathematician and researcher. Peano curve represents a fractal algorithm. The basic formula is: $\log_k N$, where k represents the primary curve, while N is the number of new-formed curves. Fractal algorithm creates from a line a Peano curve that occupies each point of that space and it stays a line (topological dimension remains 1) despite the interminable iterations. This explains how 2 meters of DNA can be "stuffed" inside a nucleus of 523.33 μm^3 volume. The fractal globule is nothing but a Peano curve conformed into a spherical shape of the nucleus. The curved line is not intersected or embrangled anywhere so if we pulled a randomly picked part, we could easily stretch it out because it would not tangle anywhere and in the same way we could put it back as it was. In the metaphase of cell division there are 23 chromosomes, respectively 23 chromosomal territories and each one makes out a small fractal molecule. Herewith, self-similarity of the fractal in the nucleus is confirmed.



Picture n. 11 Peano's curve

5.1. STABILITY OF A FRACTAL GLOBULE

Original theories suggest that the lifetime of a FG depends on the time required to thread the ends of the chain through the whole globule allowing the formation to stay knotted and also on its stringency and ability to firm its topological restrains. These restrains can be simply interrupted by DNA topoisomerase II enzymes, products of the cells enzyme activity which are able to crumple and unknot DNA chains, alleviating the replication and synthesis of proteins for which the unknotting of the double helix is crucial. There are two types: topo I and topo II. Topo II cuts the double helix by its two chains, passing through another unaffected DNA double helix and relegating cut DNA. Topo I cuts only one DNA helix in order to bind it again after the process of synthesis passes. Topoisomerase II aggravates the cell's natural homeostasis by knotting and unknotting the chromatin several times. It has been proved that increased amount of topo II added to the nucleus could develop a strong affection for topo II, during the interphase of the cell, to destroy chromosomal territories leading to the equilibration of the FG. Such amounts of topo II are able to break all the mechanisms of cytoskeleton to constrain the cell denaturation. Such constrains will be effective only in the case of enabling topo II to destroy chromatin fibers. Fortunately, topo II is not that huge antagonist in this role of the cell function. FG should be able to oppose the equilibrium long enough for the duration of single cell cycle. Another encouraging fact is that after the mitosis whole chromosomal architecture of a cell is re-established.



Picture n. 12 Equilibration of a fractal globule

5.2. MIXING AND CROSSTALK

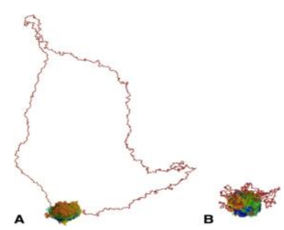
Even though the fractal globule proposes that the territorial organization of the cell can restrain the topological one there is still a major number of interactions between the chromosomal regions of the globule. Number of interactions is linearly growing with its volume.

5.3. FREQUENCY OF THE FRACTAL GLOBULE

As we mentioned, fractal dimension (fd) is a value that expresses how much space one fractal occupies. After the assumption that pirin bases are also fractaly organized as the folding process, certain experiments showed that this parameter affects the layout of the nitric pirin bases. Sequences of DNA with bigger fractal dimension had higher distribution of guanine while adenine is much more frequent in the opposite areas. It has not yet been theoretically proven why it is so, therefore additional research is needed.

6 ADVANTAGES OF THE FRACTAL GLOBULE

The measurements of a FG are very similar to the theoretical parameters of the ideal chain, as the above noted globule's size. The main advantage of fractal DNA folding is its high speed that progresses without tangling the chain and likewise uncoiling it without knotting. The gene activity is conditioned by the fast unwinding without further clutter. On the picture n.13 it is shown how for the same time the FG unfolded with no difficulties while the EG got tangled in the process. Fast untangling allows and stimulates chromatin, in fact DNA, to search faster for the most suitable place in the nucleus to transcript. Since the transcription itself is conditioned by the entry of mRNA, so it could receive the information from DNA, this unfolding allows a faster ingress of mRNA into the nucleus.



Picture n. 13 Unwinding of fractal (A) and equilibrium globule (B)

7. THE USE IN MEDICAL DIAGNOSTICS

Significant area where the concept of DNA can be applied is disease diagnosis, particularly cancer diagnostics. By applying fd on the histological microphotography of tissues we can analyze their cells. With the increase of the cells volume, their fd grows. Fractal dimension of a healthy cell is 1.1, whilst of a diseased cell it amounts 1.26 or more. Further determination showed that the higher the fd is, the cancer is more progressive and in further stage. A process called mammography based on low-energy X-rays is used in the case of breast cancer. These analyses are performed using the Minkowski-Bouligand dimension or box-counting dimension that divides the surface area into smaller squares and narrows the place that is being examined thus giving a better and more precise notion of the patient's condition and progression of the disease. It should be used in the diagnosis itself and in routine examinations for an eventual detection of the disease in early stage, as well as a faster and more efficient treatment.

8. SUMMARY

Deoxyribonucleic acid is not only one of the most refined bio-chemical complexes, but one of the greatest discoveries in the Cosmos. Yet, its structure is still very logic and everything in it has precisely determined place and function. Quite the same is its wrapping and binding, all with a proven mathematical scheme. Peanos' curve with endless number of iterations of itself is able to reassure dimensioning and shaping into any possible shape, such as sphere, revealing to the scientific world a brand new dimension of modeling, the fractal globule. This model of DNA has been introduced in 1993. and since has developed much. Forming of the globule is absolutely spontaneous and leaves no harm to the content of the same. Its lifetime is very long due to its simplified organization and allowance for DNA to transform itself. Any part of this globule can easily be unfolded without any knots left inside. Transcription of DNA is particularly being simplified in this form and that is also why we suggest FG as an ideal structure for chromatin fibers. If allowed, topoisomerase II is able to destroy this condition of DNA, reshaping it to EG which lacks of many properties of FG. At the end we emphasize the importance of fd and its use in other scientific disciplines, such as oncology, and why we should continue our research in this field.

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EXPLORING THE ANCIENT GREEKS' MYSTIC AND TOPOGRAPHICAL SYMMETRIES

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ABSTRACT

Geodesic triangulation of ancient Greece is a miraculous mystery. If we study the position of temples, sanctuaries and cities, we can see that geometrical patterns are formed such as isosceles and equilateral triangles, concentric circles passing from the same point as well as straight lines perceptibly joining cities together.
This harmonious relation comes to us, the younger ones, as a picture of colossal conception, making us shiver from great excitement.
It's worth wondering thus, if all this happened by a curious coincidence or as a result of mathematical calculations able to touch perfection. Is there a well-hidden secret refusing to be revealed even in these days?

Most of us have heard of the Parthenon, a great monument with such an architectural structure, that even nowadays it would be difficult to construct.
One of its most marvellous traits is that if we extend its four columns despite the fact that they appear parallel, they will meet up at a certain point, creating in this way a hypothetical pyramid.
The Ancient Greeks seemed to know, that in order for the monument to look stunning, the front columns should be of a bigger diameter than the others, because under the sun light they would seem smaller.
Considering these characteristics we may conclude that building a temple in ancient Greece was more than just art. It was a science.
A matter of similar complexity was the choice of location for the erection of buildings of great importance.
The location of each ancient sanctuary, temple, oracle, monument or even city was not randomly chosen, but all the locations seem to be related in a certain way.

B. How marvellous is that?

Many of you may have heard, that the aforementioned locations of importance for the ancient Greeks, were chosen carefully so that they would shape hypothetical geometric shapes, like triangles, polygons and concentric circles.
One cannot but be surprised when realising the extent of this phenomenon, which is called geodetic triangulation. What we are really talking about is the existence of a geometric geodetic network!

What is geodetic triangulation?

"Triangulation" generally means the coverage of a certain area with triangles, while geodesy, as derived from the Greek word "geodesia" means "land split up", so geodetic triangulation actually means the splitting up of land in triangular parts.

Geodesy is one of the oldest sciences to date as are maths and astronomy.
Aristotle, 4th century B.C, was the first to refer to the term geodesy and describe it as "the art and the science of calculations for the division of an area"

According to Greek mythology, the location of a city or a temple was determined by the gods.

Aristotle wrote that the temples should be properly located. The monuments found in agricultural lands' should be symmetrically distributed and devoted to gods and heroes.
Another ancient Greek geographer and historian, Stravon, wrote that those who undertake the construction of such buildings should consulted with astronomers and geometers about the shapes of the buildings and the distances between them.

Some say that the locations were also chosen in a specific way, so that they would be in harmony with shapes which depict constellations.

Geodetic triangulation in ancient Greece and positions of temples and monuments are a mystery and two questions arise.

Were the locations chosen following this motif, everywhere in the then known Greek land? The most important question is, whether this phenomenon was premeditated, carefully following a plan or was it just an impressionable coincidence?

There are many references of geodetic triangulation in ancient scripts, but the whole thing seems to have passed as an unnoticed incident, since there are many more achievements to surprise this planet.

Jean Rissen, in 1967, was the first one to study the harmony and the beauty of the interconnection among Greek ancient temples. He tried to verify that ancient Greeks really liked to play mathematical games on the map.

Right after that, Theofanis Manias, a brigadier of the military air force, also observed the phenomenon by studying relevant references from many military pilots. He noticed that the airplanes consumed the same amount of fuels when covering distances of triangular shape and when he checked those routes on the map, he noted that these triangular routes were shaped by ancient temples.

That was the trigger for him to further study the internal mathematical and astronomical harmonization of the monuments.

The proof of these assumptions came later with the use of GPS technology and geographical pinpointing as prior to this, it was difficult to calculate distances using only compasses and rules. Later on, several measurements of temples and monuments by Professor Markatos proved that a unit of geodetic triangulation was probably an isosceles triangle. This also confirmed the existence of symmetries, formally proving the existence of triangulation. GPS calculations also showed that some of the triangles were indeed isosceles and the deviation in the lengths of each side was only some meters or in some cases merely some centimeters!

Before the presentation of the existent shapes on the selected areas, we should review the towns we are going to talk about.

We should also mention that the formal unit for distance measurements back then in ancient Greece was a stadium or stadion. One stadium is equal to the length of an athletic stadium, about 600 feet or 195,15 meters.

We will now, we present some information about the places we will examine in the present study.

Deplhi

Deplhi is a region in ancient Greece, where one of the most important oracles is located. This town gradually expanded and became one of the most important and sacred towns of ancient Greece.

Eleusine

Eleusine is a town near Athens. In ancient Greece, Eleusine, Athens, Olympia, Deplhi and Delos were the five sacred towns of Greece. The Sacred Road ended up in Eleusine.

Olympia

Olympia was the most famous sanctuary in ancient Greece and was devoted to Zeus. The Olympic Games took place there. Inside the glorious temple of the god, there was a statue of Zeus made of gold and ivory. It was constructed by Phidias and is considered one of the Seven Wonders of the Ancient World.

Iolkus

Iolkus was an ancient town, located to the north of Athens. It is mentioned in Homers' epic poem, the Iliad, but also by other writers, such as Isiodus and Euripides.

Ephesus

Ephesus was one of the most important towns in Asia Minor, on the Aegean coast. A famous temple devoted to goddess Artemis was built 1 mile (10 stadia) from the town.

Idaean Anand

Idaean Anand is a cave located on mount Ida, in Crete, where according to Greek mythology, Zeus was born and raised. Greek mythology includes a lot of myths about the guardians of the cave.

Knossos

This is nother important town in ancient Greece, located in Crete. It was the hometown of the wise king Minoas and the center of minoan culture.

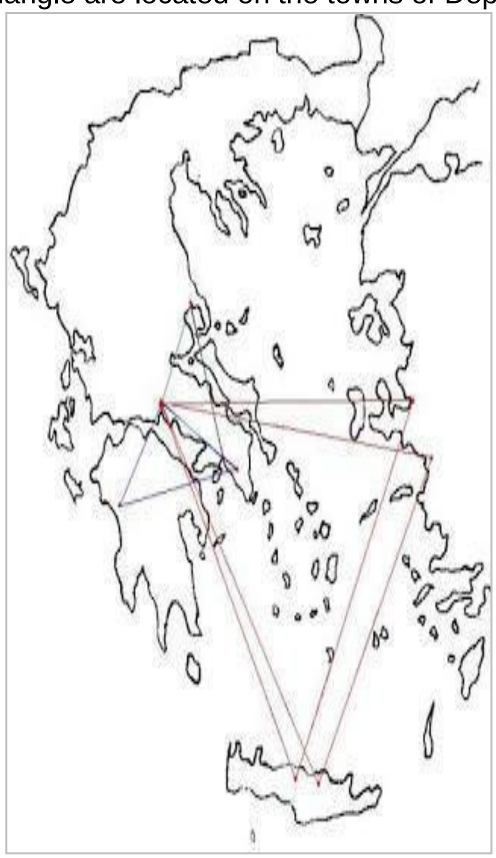
Smyrna

Smyrna is one of the oldest towns and ports in the Mediterranean Sea. It was established in 3000 B.C. and still exists today.

Dodoni

The first references to the Dodoni oracle as a sanctuary date back to 2600
B.C. It is the oldest oracle in the broader ancient Greek area.

- So the distance between Deplhi and Athens is 660 stadia and the distance between Deplhi and Olympia is also 660 stadia. We can see that an isosceles triangle is formed and its vertices are located on the towns of Deplhi, Athens and Olympia.
- Similarly the distance between Delphi, Eleusine and Iolkus is 550 stadia respectively. Here is another isosceles triangle with its vertices located on the towns of Delphi, Eleusine and Iolkus.
- One of the most interesting triangles is the one with its vertices located on Delphi, Idaean Anand in Crete and Smyrna. It is an equilateral triangle and the length of each side equals 2198 stadia. The vertices of the triangle are located on the towns of Deplhi, Knosos and Ephesus.



But it is not only the existence of triangles that is mind boggling.
The straight line that links the island of Thasos with ancient Olympia goes through Delphi. There is a similar line that links Dodoni and the island of Kasos and the straight line that goes through the towns Delphi, Smyrna and Chalkida is parallel to the equator.

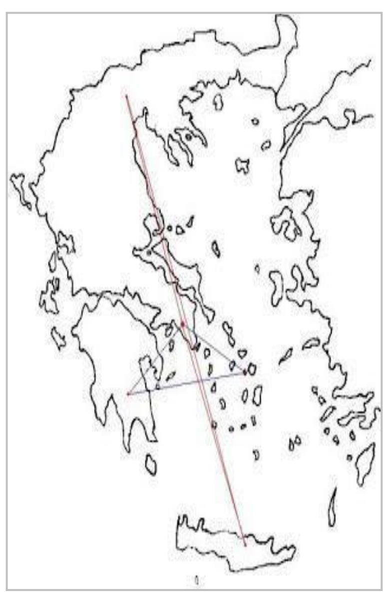
Athens

The history of Athens goes back to 3200 B.C. Ancient Athens was a powerful city-state, center of arts, knowledge and philosophy. Plato's academy and Aristotle's Lyceum were there.

Delos is a small island part of the Cyclades island group. It became an important religious but also commercial center.

Sparta was a city-state in ancient Greece, located on the eastern Peloponnese. Sparta remains famous through history because of its great military power and its great number of slaves.
The distance from Athens to Sparta and Delos is 800 stadia, forming an isosceles triangle with its vertices located on Athens, Sparta and Delos, aw mentioned before.
Athens is also of equal distance from Knossos and Pella by 1765 stadia and in this way another isosceles triangle is formed, with vertices on Athens, Cnossos and Pella and the angle between the equal sides is somewhat less than 180 degrees.

Eleusine is located:
100 stadia away from Athens. That is the same distance as that between Eleusine and Megara
- 330 stadia away from Corinth. That is the same distance as that between Eleusine and Sounion
- 220 stadia away from Amfiarius. That is the same distance as that between Eleusine and Marathon
- 1700 stadia away from Pella. That is the same distance as that between Eleusine and Smyrna
- 1815 stadia away from Pergamon. That is the same distance as that between Eleusine and Miletus but also the same as the distance between Eleusine and Knossos.



Sparta

At the dawn of the 4th century B.C. the capital of the Macedonian state was transferred in Pella and evolved in to a political, economic and cultural center, where Alexander the Great was born.
Zakros is a village on the east coast of Crete and remnants of ancient minoan Zakros still exist today. Ancient Zakros was one of the four centers of the Minoan civilization.
Sparta is located at a distance of 1700 stadia from Knossos and Dodoni respectively, forming an isosceles triangle with vertices on Sparta, Knossos and Dodoni
Furthermore Sparta is 2200 stadia from Pella, but also from Zakros. Once again we can see an isosceles triangle forming with its vertices on Sparta, Pella and Zakros.
Another isosceles triangle is Sparta, Delos and Iolkus. The vertex is Sparta and the equal sides are approximately 1375 stadia.

Argos

Argos is a town in the Peloponnese and was one of the most famous centers of the Mycenaean culture.
The distance of Argos from Athens, Deplhi and Olympia is exactly the same! The triangle with vertices on Athens, Delphi and Olympia is isosceles and the equal sides measure up to 550 stadia.

Delos

The asklepion of Epidauros, a healing sanctuary, was the most famous asklepion and was visited by people from all over the Mediterranean area in order to recuperate from an illness.
The asklepion of Kos was another well-known healing sanctuary, while nowadays it is one the most famous monuments of the island. Back in ancient Greece, it was a temple but also an educational faculty for medicine. Hippocrates used to teach there.

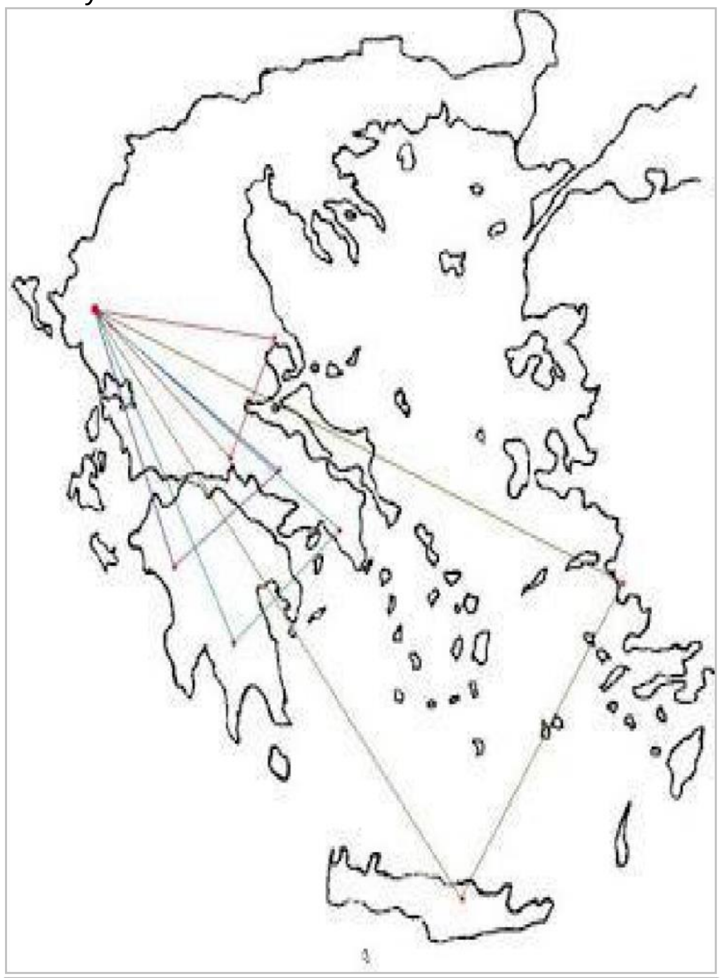
Pergamon was a glorious and rich city in Asia Minor and was the capital of the kingdom of Pergamon.
- If we link the location of the Epidauros' asklepion with the one in Kos and the one in Delos, then again a triangle is formed. The equal sides are the distances between Delos and Kos and Delos and Epidauros respectively and the length of the equal sides is 1020 stadia!

- Another isosceles triangle can be observed among Delos, Smyrna and Thebes.
- Similarly, there is another isosceles triangle among Delos, Sparta and Pergamon and the length of the equal sides (Delos-Sparta and Delos- Pergamon) is 1360 stadia.

It is also located:
- 1080 stadia away from Idaean Anand. That is the same distance as the one from the Trophonius Oracle.
-1460 stadia away from Delphi. That is the same distance as the one from Alexandria Troas.
- 1530 stadia away from Rhodos. That is the same distance as the one from Figalia in the Peloponnese.
- 800 stadia away from Athens. That is the same distance as the one from Kardamili in Chios.
- 1256 stadia away from Rethymno. That is the same distance as the one from Knossos

- 1188 stadia away from Corinth. That is the same distance as the one from Mytilene
- 1859 stadia away from Samothraki. That is the same distance as the one from Thermon.

-1859 stadia away from Mycenae. That is the same distance as the one from Argos.



Dodoni

The Trophonius oracle was one of the least famous during the ancient years, but also very unique as it does not look like any other oracle. It was located 130 km northwest of Athens.
Miletus was an ancient town in Ionia, built on the west coast of Asia Minor and was inhabited since the Copper Age.

- It is of equal distance from Iolkus and Delphi by 1050 stadia
- It is located 1210 stadia away from Olympia and the Trophonius oracle
- Another isosceles triangle of interest is the one with its vertices on Dodoni, Athens and Sparta. The length of the equal sides (Dodoni-Athens and Dodoni-Sparta) is 1700 stadia
- The biggest triangle observed is the one with a head vertex on Dodoni with equal sides of 3300 stadia each that reach Knossos and Miletus.

Knossos

Mycenae was an ancient city located almost 90 km southwest of Athens. It was one of the greatest cultural centers at the time.
Epidauros is a historic town the same region, in the Peloponnese. According to mythology it was also the birth place of Asclepius. Its strategic location, but mainly its asklepion contributed to the town's growth.
Knossos is equidistant from Sparta and Epidauros (1700 stadia)
It is also equidistant from Argos and Mycenes (1815 stadia)

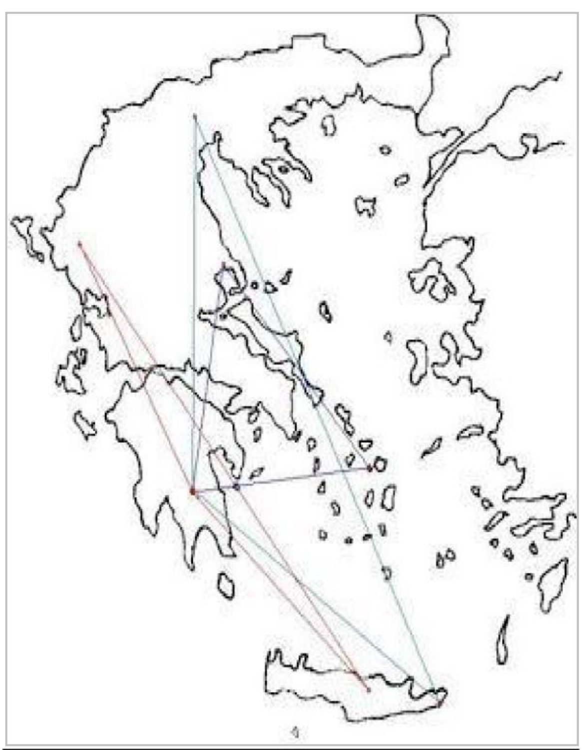
Pella

Samothraki is an island located on the northeast part of the Aegean sea. The island is known for its ancient Greek statue of Nike (goddess of victory), which is exhibited in the Louvre, Paris
Alexandria was an ancient Greek town on the coast of Asia Minor.
Alexandria ad Issum or Alexandria Minor, located in Kilikia, Asia Minor was also a ancient Greek town.

- Pella is equidistant at about 1360 stadia from Deplhi and Samothraki
- It is also equidistant from Athens and Alexandria Troas.
- Its distance from Paros, but also from Smyrna is 2350 stadia

- Similarly its distance from Alexandria in Egypt and Alexandria ad Issum is 6600 stadia.

Could we safely suppose that Alexander the Great established two different cities at a same distance from the capital of Macedonia?



Some more noteworthy comments

- The isosceles triangle formed by Dodoni, Olympia and the Trophonius Oracle is part of a regular decagon. The properties of this decagon can be extended and meet Ilion, Smyrna, Knossos, Larisa Troas, Sparta, Paros, Phestus and other cities.
- The isosceles triangle formed by Dodoni, King Nestor's Palace and Eleusine, with a top angle of 40 degrees is part of a regural nonagon
- The triangle of Dodoni, Athens and Knossos is part of a regular tridekagon
- The triangle of Dodoni, Knossos and Miletus is part of a regular dodekagon with a top angle of 30 degrees
- The isosceles triangle of Dodoni, Delphi and Iolkus is also part of a regular dodekagon
- The same observation is made for the isosceles triangle shaped by Dodoni, Olympia and Trophonius oracle
- The distance of Chalkida from Athens and Sounion is the same. Similarly Deplhi are located at the same distance from Olympia and Athens.
- Starting from the center of the Parthenon, if we could join some points, like Theseion or Theseum (Temple of Hephaestus), Pnyx, the base of Filopappou Hill and the center of Olympian Zeus temple, an octagon would form, the sides of which would be equal to the length of the Parthenon multiplied by seven. (wow)
- If the Parthenon columns were extented, they would meet at a height of 1852 meters, forming a pyramid of half the volume of the Great Pyramid of Egypt…

Someone may say that all of these are just coincidences and that it is only natural for such correlations to exist. Possibly the found proportionate correlation of so many important points with each other might be characterised as a really far-fetched notion, but it surely is a phenomenon that is met too often, to just accept that it is a coincidental occurrence.

So what conclusions can we draw?

Nothing more and nothing less than the fact that in ancient Greece the area for the erection of buildings of importance, such as sacred temples, towns and cities were carefully chosen.
All of these are still inconceivable even for modern scientists. It is actually noteworthy how they were able to measure distances with such precision. Also who inacted the "Law" that defined the adoption of certain methods and criteria for the choice of the final locations.
But of greater interest is why ancient Greeks decided to follow this system? Is there a special meaning attributed to the chosen areas?
The last question offers of course great opportunity for speculation.
Were the chosen place full of some kind of terrestrial energy? Or maybe it was just a symbolic choice in order to conjugate and unify sacred places, assuring, according to their beliefs, harmony with the whole universe? What else?

Despite all of the above, one thing is for sure. The wisdom of the ancient Greeks seems to surpass even our wildest imagination.

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MATHEMATICS AND MUSIC CAN EXPLAIN EVERYTHING

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ABSTRACT

Mathematics and music are two sciences which are strongly interrelated. These two arts have been affecting each other since ancient times and this interaction comes to our days. Music is a way of life and expression. Through music you can "travel" by carrying melodies in your inner self and in this way you can feel the beauty filling your soul and spirit.

Music therefore, is a way of expression that is common to every person. In parallel, the beauty of mathematics, for those who can comprehend it, lies in the simplicity, the symmetry and the elegance of their formulation. For Plato, mathematics was presumed "the utmost culmination of beauty". In addition, mathematics is undoubtedly a universal communication code.

When we seek for interrelations between music and mathematics, we reach the conclusion that MATHEMATICS AND MUSIC CAN EXPLAIN EVERYTHING.

No harmony would exist if numbers were not in existence. There would be no harmony if no man existed so as to listen to it and reckon it as such in order for numbers to be the appropriate tools for it.

No harmony can exist by itself.

Is it mathematics of music or music of mathematics that affects the human brain and activates it?

Introduction

Math and music are closely related on a scientific basis. Since ancient times, an interaction between them was observed and that observation still applies nowadays.

Most people think of math as an enigma. The science of math is characterised by a vague perception of numbers and calculations, as taught in school and is usually followed by indifference. Most people think of math as a cold and "lifeless" subject.

Music on the other hand is about feelings and emotions, it is about life. Everybody has sang at some point of his life or attempted to even play some music or just whistle a catchy rhythm.

Therefore, the relationship of these phenomenologically irrelevant sciences may not be obvious, but is surely worth discovering it, since they are truly closely related.

The first bad impression that math make, always change when someone learns about Pythagoras or about the applications of Johan Sebastian Bach.

What to refer to first? Fractions, quotients and proportions found in musical intervals?

Fibonacci sequences are found in musical synthesis but also used on musical instrument construction?

Scale, trigonometry and the wave form of sound?

In order to answer these questions, we need to investigate the mathematical relationship between music notes and the reason for the observed relationship.z

What is really interesting is that those relationships arise from nature itself and are not human contrivance.
Humans did not construct music, but it was eavesdropped and taught to human by nature.
The aim of this study is to investigate our aesthetic criteria about music as explained by math.
Structural elements of music are the sounds we call notes.
The notes were discovered long before music theory developed and of course they were only chosen not for a particular reason, other than that they just sounded nice, so our ability to listen and compose music is more congenital rather than acquired.
Sound are actually vibrations transferred through air and when these vibrations meet an obstacle they cause a phenomenon called resonance.
That also means, that when listening to music, our ears are not the only organ participating in the process but our whole body resonates with the vibrations and this happens even with closed ears. We understand that resonance might be the key for our congenital ability to "feel" the music, as we usually say.
Returning to the math-music relationship one can easily note while reading a book about music theory, that it seems like reading a difficult math book.

How is music related to maths?

Pythagoras, a mathematician and philosopher, was the first one to study musical sounds. He was well aware of the relationship between music and numbers. Scientists believe that Pythagoras and his students deepened their knowledge on the subject by studying the ancient musical instrument "monochordon", an ancient instrument on which Pythagoras used to study the music notes in relation with the length of the string, which produces each note.
This instrument in its simplest form consists of one string, streched over a box, which works as a speaker. The produced sound depends on three factors:

1. The string thickness
2. The traction force of the string, i.e. the curling. The more the string is pulled, the more acute is the sound emitted (sound of high frequency)
3. The length of the string. A short chord produces an acute sound, while a long one produces a bass sound (a sound of lower frequency)

Pythagoras made an observation. A 90 cm string produces a specific sound. The sound with the greater affinity to the first one is produced by a 45cm string, i.e from a string of half the length of the first one. Following the affinity series of the sounds, the next more similar sound was produced by a 60 cm string, this is two thirds of the length of the first string.
The glide up technique (which means the use of mobile frets) allows to change the string length and therefore the tone of the sound. From the string length ratios derives the mathematical relationship among sound tones.
For example, if the fret is located in the middle of the string, the mathematical relationship occurring is expressed with a ratio 1:2, the well known in the music word octate.
Likewise, when a fret is located in a position corresponding to the 4/5 of the string length προκύπτει το διάλυμα μεγάλης τρίτης…
Following this path of thoughts, lead him to observe that a mathematical normality is transformed to music and vice versa and set the ground for further investigation of the relationship between math and music.

Pythagoras noticed a constant relationship between the string length of a lyre and the basic chords (1/2 for the 8th chord, 3/2 for the 5th and 4/3 for the 4th one)
When two sounds combined produce a pleasant tone, then the string length ratio (from which they are produced) is an integer number and those sounds are called harmonic.
This wonderful trait of the harmonic relationships is due to the fact that they contain the first four natural numbers (1,2,3,4), the sum of which is 10, the sacred number of Delphi.
According to Plato and Aristotle music is not just art, but music in its core is math. Their writings about music look like math or geometry textbooks.
Plato said that there are four subjects which cultivate human spirit. These are music, math, geometry and astronomy.
The aforementioned related sciences, can be described as Math: stationary numbers
Music: numbers in motion
Geometry: stationary multidimensional units Astronomy: numbers in motion
Many mathematicians tried to calculate musical intervals, like Archetas worked on the ratios of tetrachordon spaces.
Eratosthenis defined the difference between major tones (9/8) and minor (10/9) tones.

Maths, music and the golden ration

The math-music relationship is not only limited to ratios and analogies of music intervals.
Fibonacci numbers, 1,1,2,3,4,5,8,13,21,34,55,89,…
Each one of them is produced by summing the two prior numbers.
Those number display an interesting property.
If two sequent numbers are divided with the bigger one as a numerator, the result comes any time more closely to 1,618, which is the golden number φ.
What do all of all these have to do with the music scale? Indeed, they have.
A piano scale has 13 keys, 8 of which are white and 5 are black, while the last of them are divided into themes of three and two keys.
In western culture, music notes are based on Fibonacci series, since the frequencies of the tones are related to these numbers.
Musical compositions often reflect Fibonacci numbers. The dynamic golden ratio, besides other various application, is also used in music to produce rhythm changes or to develop a melodic line.
Main characteristic of many compositions is that their structure is divided in two parts either in a symmetric way or according to the golden ratio.
A famous example is the song "Hallelujah" composed by Handel. Furthermore, the golden ration is applied on every Mozart sonata.

TRIGONOMETRY-FOURIER

Sound is transferred through waves.
The sound produced by a tuning fork (diapason) can be described by sinwave.
This sinwave is the graph of the function $\psi=\alpha\sin\beta x$, where α and β are positive numbers which define the wavelength and frequency when x can take any value from a 360 point interval.
But what can math tell us about more complex sounds and how could the well sounding of some sounds or the irritance some others cause be explained?

The graphs describing all sound, including our voices, follow a normality, which means that the displacement-time graphs reoccur precisely many times per second.
Sound that are characterised by periodicity are generally pleasant and technically called music. Fourier further clarified this topic by stating that every periodic sound is the sum of various sinwaves of the above mentioned form and the frequencies of these equations are integer multiples of the lowest frequency.
We conclude that any complex sound is just a combination of simpler sounds like those produced by tuning forks (diapason).
It is now more than obvious that every sound can be mathematically described. Even the most complex sound, in math it is just a sinwave function.

KEPPLER: THE MUSIC OF PLANETS

The music of the stellar globes was first mentioned by Plato, 2500 years ago. He used to call music as the beauty of the universe and also said that "Music is the sound motion in order to reach the soul and teach us virtuousness. Music is a moral compass. It gives soul to the universe, wings to our thoughts, lifts off our imagination, gives joy to sadness and life to everything".
Prior to Plato, Pythagoras had reached such a level of sensitivity that was able to listen to skies symphony, to the music of the stellar globes.
Music reflects the harmony of the skies. The harmonic relationships of the numbers can be met at the planets. The planets produce various music sounds, while rotating, sound that cannot be heard.
Keppler studied the scripts of ancient Greeks, mainly Plato, and concluded that every planet emits a different note while in its orbit.
Some of these sounds are similar to the sounds of dolphins, waves, birds, wind…
The sounds that arise from the planets are closely related to the seven music notes.
The correspondance of the planets with the produced notes are

1. Sun=E
2. Venus=F
3. Mercury=G
4. Moon=A
5. Cronos=B
6. Mars=D
7. Zupiter=C

In his work De Harmonic Mundi he designed a partiture, which depicts the planets to perform their universal concert.
This way a book about the music of the skies was written.
……Περίεργη σύνταξη

Bach

The european music relied on the principles of harmony of the pythagorians, until Johan Sebastian Bach composed "The well-tempered clavier" and suggested the division of the octate into twelve semitones. This was also prosposed by Aristoxenos 2000 years before Bach.

Beethoven

How was Beethoven able to compose such masterpieces while being deaf? Once again math can give us some answers.
Natalya St. Clair used the famous "Moonlight Sonata" in an attempt to explain how Beethoven could have expressed such feelings with some help of mathematic precision.
The answer for his achievement can be found on his partitures.
"Moonlight Sonata" starts with a steady slow rhythm consisting of triplets of notes.
One and two and three and one…
Despite the fact that, the notes seem simple, every triplet of notes contains an elegant melodic structure revealing the extraordinary relationship between music and math.
Beethoven once said "I have always had a vision in my mind when composing and always followed that vision"
Imagine an octate piano with 13 keys και καθένα διαχωρίζεται από ένα βήμα
A basic scale or a typical secondary one uses 8 of these keys with 5 whole intervals and 2 ….
Beethoven added the non defined spectres of emotion and creativity and of course a little math.
Δεν καταλαβαίνω το ελληνικό κείμενο.
So, although we can see the mathematical concept in the partitures we still cannot understand why this music touches the heart of the audiences. Beethoven's intelligence lead him to compose without even being able to listen to the music. In a similar way, musicians feel the math and mathematicians think of the music. Music is the dream and math is the work in our lives.

NOWADAYS

Nowadays many musicians use math to compose. A main representative is Iannis Xenakis.
Xenakis is a pioneer in algorithmic music composition. He has also developed a theory about digital composition based on sound production using mathematical functions.
Notworthy is the fact, that he is also interested in the relationship between math and music, in order to find out how "The art of fugue" (Bach) can be
described by mathematical models, so that the music can be depicted by graphs like a visual representation of music.
He tried to apply the laws of physics to music.
The first work toward this pioneering direction is called "stochastic music"
This is the first work, which includes audible sets that consist of various single sounds.

Xenakis also uses a variety of mathematical theories, like:
- Set theory, as applied on his work "Erma". The whole length of a piano is considered as set A including 3 sub-sets. The result is a variety of sounds.
- Game theory. When this theory-technique is followed, two conductors react to each others' choices.
- Bull algebra combined with Set theory. This theory is used by Xenakis in his works "Erma" and "Eonta" and is called "symbolic music" by him.

What are the benefits of music? We refer only to some of them.
- music helps speech development
- helps children learn math. When children learn about rhythm, they basically learn about quotients and fractions
- music promotes socialization
- music promotes mental growth of children as it enhances neural system and brain activity
- music encourages self-expression and self-esteem, while commuting plenty feelings and offers shy children or children with other speaking difficulties a way to express themselves.

BRAIN-MUSIC-MATHS

Let' see how music and math influence the human brain.
When listening or playing music the right cerebral hemisphere is activated.
When learning to study music, to understand the music and the partitures the left cerebral hemisphere is activated, more specifically the same area that is in charge for analytical thought and mathematical thought.
A 1997 study suggest that listening to music arouses the brain neurons, which determine the spatial-temporal reasoning.
This kind of reasoning is used in higher brain activities such as those in which we engage while doing math, playing chess or composing music.
Furthermore, studies have shown that children that study music from an early age have better math skills. This happens because:
Recognition and understanding of basic numbers is necessary in both music and math
Problem solving skills are required in both activities
Both activities require an ability of motif recognition, which promotes mental growth of students
Motifs are structural constituents of mathematical knowledge
If a musician was asked about the common ground between Bach and Joplin, would surely look like he would have seen an alien. What kind of relationship could there be between a master musician , favourited by the church and the Woodstock priestess Janet Joplin?
Brain scientists say:
" The input of audible stimulus is actually located on neurons at the upper part of the brain lobes, which are called aesthetic neurons and when a chord is harmonic and pleasant the ratios between its frequencies are equal to small natural numbers."
In a relevant study, 1800 music tracks were analysed, covering as a sample 4 centuries of western music and it was found that all of them consisted from a repeated rythmic motif, an "audible fractal". Because of this motif, the music of Beethoven, Bach, Haydn and Mozart were of such great resonance, similar to that of the song "Summertime" of Joplin.
We may say that this motif is an emotional code, hidden in our brains and when aroused, causes vibrations to our souls.
Physicist Shaw, who studies brain activity, says that music plays a pedagogical role in math. He suggested that those who listen to the sounds of Mozart seem to have better mathematic abilities and logical reasoning.
Is there a relationship between neurons and harmony?
In 1978, the first brain neuron model was produced. Shaw corresponded music notes on this model.

He found himself surprised, when he listened to an harmonic music as a result and not some unclarified and irrelevant sounds. Based on this occurrence he made the assumption that music generally improves mental activities.

The first official study towards this hypothesis, in 1993, showed that students who took part in logic exercises, achieved better scores after listening to a particular Mozart sonata. The increase of the observed scores was about 35%. The phenomenon was later called 'The Mozart effect"

We should keep in mind, that:

Music, just like as math, is part of our everyday lives although this is not always actively perceived.

Music related education from an early age seems to modify brain structure.

It is commonly accepted that math is not always a pleasant subject for students, while it is often characterised as a difficult topic

Rhythm and emotions are independent of the place and the time that goes by and so they remain unalterable and are perceived in a natural way.

Nowadays science and technology tend to keep us remote from nature, some may say they even make us miserable. Art and especially music may become a forgotten mean of communication between us and our true nature.

Maybe we should have listened Pythagoras calling:

"The whole world is numbers and music is its beauty, didn't I told you so?"

MATHEMATICS AND POETRY: AN UNDOUBTEDLY CHARMING ENCOUNTER

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ABSTRACT

As Nobelist Theoretical Physicist Heisenberg once said:
"...There are only two languages with which man can describe nature; Mathematics and Poetry"
Do mathematics and poetry have common characteristics, similarities, some relation, or are they separated by a vast gap as many believe?
Poetry is not only a way of expressing emotions, but also an attempt to interpret the world. It is man's general perception of beauty, which becomes the cause of inspiration for all areas in which it is expressed.
Mathematics is the science concerned with quantity (meaning numbers), structure (that is, shapes), the connection between all countable things in reality and imagination.
- Is there such thing as a relationship between mathematics and poetry?
- How can mathematics and poetry bring together logic and emotion?
- How much and to what extent is our brain influenced by the poetry of mathematics and the mathematics of poetry?
- Could, perhaps, mathematics and poetry be the two windows which we need to open in order to gaze into and realize the chaos of reality?

Foreword

The relationship between mathematics and poetry is not something known to exist by many. Few are the people who realise how closely related they truly are. As one explores these two concepts, they begin to understand in how many ways one supports, or even completes, the other.

What are mathematics?

It is the science that researches all matters concerning numbers, shapes and the relations between all countable things.
Mathematics research structures which either come from natural sciences or from mathematics itself. In school, for example, maths study quantities, shapes and distances. This is done by using productive logic and then compiling the results in systems, axioms, theorems et cetera.

What is poetry?

It is art. Its roots are the expressive capabilities of language. It is musical thought, thought that expresses our inner world.

Interaction between mathematics and poetry

Mathematics and poetry constitute two different areas of human spiritual activity which develop in different ways, but serve common, individual or otherwise, goals and needs. Each of these spiritual activities has its own tools and means of expression.
The interaction between them is completely natural. It stems from the fact that mathematics is based on our need to logically describe the world, while poetry approaches things through emotion. In the end, both give us a better understanding of ourselves, since mathematics delve deeper into the human consciousness and poetry further explores the human soul.

More than often, poetry uses mathematical terms to describe situations, feelings, characters. On the other hand, mathematics only use patterns to help us memorize things. Besides, it is a fact that the algorithms that most easily stay in memory, are those that have a rhythm, a pattern.

We are undoubtedly talking about two ancient creative and spiritual human processes. People developed myths in order to explain the surrounding world. They created myths about the birth of the world, myths about gods, myths filled with mystery, all based on man's awe about all that he could not explain.

And on the other hand, mathematics was a useful tool, even for the primitive man. There had to be a tool for measuring time, days, the phases of the moon. Through the ages, all this became what we know as science, which attempted to explain nature and myths through logic.

Poetry and mathematics answer the three questions that always bothered humans.

1) What is our position towards nature?
2) What is our position towards society?
3) What is our position towards death?

It becomes clearer that the common points between mathematics and poetry are not so much a fiction, as one may immediately conclude.

Additionally, great mathematician Hardy writes in his biography:
«The mathematician, just like a painter or poet, is a designer. Their designs are made of ideas. The painter designs using shapes and colours, the poet uses words, instead. In poetry, too, ideas count greatly.»

Mathematics used in poetry

Through the ages, countless poets have attempted to express, define and elaborate on maths through their poetry. Besides, let us not forget that the structure of most poems is based on metres, syllables and sequences, through which we can discern mathematics. While the content of a poem may vary, its structure will always be purely mathematical business.
Most poems are in an alignment. By the term alignment we mean the verses, lines and metre of a poem.
Initially, let's take a look at what these terms exactly mean.
Verses can be defined as the "paragraphs" of a poem, and are made up of lines.
Lines are, well… the lines of words of a poem. These words, in turn, are constructed of syllables.

The rhythm in which we pronounce and entone these syllables is called a metre.
All poems have a number of syllables and verses. That way, the poet kind of "builds" on a mathematical motif.
In some cases, poets will write a poem intended to have a geometrical structure as its main feature.
In a poem written by Lewis Carroll in his book "Alice in Wonderland"m the words had been carefully picked in order to fit into the shape of a mouse's tail.
A poem structured in such manner, to resemble geometric shapes, are called visual poems and are one of the many examples of the use of mathematics in poetry.

In other cases, poems are written with a strictly defined number of syllables and verses. For example, the haiku poems, of Japanese origin, almost always consist of 17 syllables in 3 verses. The alignment is as follows: Seven syllables in the first verse, five in the second and seven in the third.

Ever since Homer's age, up until ours, many poets insert numbers and mathematical terms in their works. It is also noteworthy that many mathematicians, of various origin, write poetry.

It is not rare for people to decide and work on both mathematics and poetry simultaneously. Let us not forget that in ancient Greece, important people from the area of mathematics used to study more theoretical sciences as well, such as literature, music and rhetoric.
Accordingly, in Arabia, scientists usually owned the title of poet, astronomer and mathematicians at the same time.
Subsequently, many poems decide to express their thoughts on mathematics through their works, by comparing science with a more "romantic" side of life, or by describing how mathematics are everywhere in our daily life.

This way, a new form of poetry was formed, lyrical mathematics. The one who introduced this term was nobelist poet Odysseas Elitis, whose poem "The way of thus" revolved around a very simple and understandable term, "thus". This term is used to show that we have solved an equation or mathematical problem. Elitis compares the
proccess of solving a problem with life. This way, he concludes that the secret of life is always reaching a "thus", while always having the right answer.

I will now attempt to analyze for you some poems that revolve around mathematics in our life.

- Odysseas Elitis, in his poem «Little Sailor», compares the sea to a school. In the poem, he explains how this unusual school taught him two very basic mathematical functions. First, Elitis explains mathematical analysis. He analyzes his country, Greece, to three parts: an olive tree, a vineyard and a ship. Subsequently, he states the very simple operation of addition. By combining the aforementioned olive tree, vineyard and ship, the result of the addition would be, of course, Greece.

- In «Six Significant Landscapes», Wallace Stevens talks about rationalists.

  Rationalists, wearing square hats, Think, in square rooms,

Looking at the floor, Looking at the ceiling. They confine themselves To right-angled triangles.
If they tried rhomboids,
Cones, waving lines, ellipses --
As, for example, the ellipse of the half-moon--

Rationalists would wear sombreros.

In this poem, Stevens wonders about rationalists' "rigid" thinking. The square shape usually resembles something sturdy, something rigid. He claims that, were rationalists to change their confined views, they would trade in their square hats for… sombreros. As we know, sombreros have swirly shapes and elliptical lines. With this analogy, the poet associates open-mindedness with bent shapes.

- Next on, the poet Ector Kaknavatos, who was one the most genuine supporters of surrealism during his time. In one of his poems, he describes a city that is being pillaged by an unknown force. To our great surprise, however, this force is not a hostile army, but an equation. An equation is attacking the city. Kaknavatos aims to show how mathematics can shake us up. He additionally describes the equation as being dressed in iron armor. With this, he attempts to show how challenging, yet important, finding the solution can be.

- One specific mathematician did a great amount of work on poetry as well. Manolis Kseksakis. He used to say that poetry and mathematics were two sides of the same coin. He wrote a whole collection of mathematical problems in the form of poems. For example, «The Horseman»

A horseman rides with 12,5 km per hour
And hunts a pedestrian who started walking
Many years ago, and still is
After how many hours will the horseman reach him
And at what distance from the starting point
Will he murder him?

The poet begins by calling a very simple problem. A horseman pursues a pedestrian, who started before him. We are tasked with we are tasked with figuring after how many hours the horseman will reach the pedestrian, and at what distance from the initiation point the horseman's purpose will be fulfilled.

- Finally, the poet George Vafopoulos, wrote an amazing poem called «The great Cone». In this poem, the world is depicted as a purely mathematical model; the cone. This cone is ran from bottom to top by a spiraling line, resembling the life of a person, the course which they follow. Childhood is located at the bottom, on the big spirals. During these, the person slowly forms and matures, but still has quite a small line of sight. By living life, moving up the cone, the spirals get smaller and smaller, but the line of sight gradually enlarges, until the person has reached the top of the cone.

The different meanings of numbers in poetry

Numbers, the base of mathematics and all other sciences, have been troubling philosophers and poets forages. Through poetry, the supernatural properties of numbers are expressed.
The number zero has been a source of inspiration and thought for many poets and is regularly featured in poetry. It usually represents nihilism, complete nonexistence, but sometimes represents existence itself.
The "magic" number three is considered by many to be the perfect number, due to being the first number to have a beginning, a middle and an end. This property has given it a special position in poetry, as it is regularly used to highlight important persons and situations.
Last but not least, the number seven also has a crucial role in poetry. It mainly symbolises the seven colours of the rainbow, but additionally has great religious value.

Differences between poetry and mathematics
Even though there is no denying their similarities, it would be a lie if we claimed that poetry and mathematics are the same.
Their differences are what make their encounter so enthralling. Poetry is an ancient way of translating our experiences. It is a form of communication that aims directly at our emotions.
Human curiousness, however, always demanded a more intricate, more "technical" means to describe the world. That is how sciences were created. They narrate life, not with words, but with the mathematical language of nature.
Let's not forget that they are two different means for the accomplishment of the same task. Writer Antonio Tabucchi, in his book "Tristano dies", explains that geometry conflicts with chaos and that's why humans created windows, to confine the outside world into a rectangle. It is thus necessary to open windows, in order to understand reality and uncover its secrets.
Could mathematics and poetry be two such windows? Perhaps.
Or even better, could they be one single window with two different angles, depending on how one looks at it? Two angles that interact with us in the aforementioned ways. Through emotion and through logic.

How mathematics and poetry affect the human mind
A shared attribute of poets and mathematicians is that they help us put chaos in order, through their work, they change our view of things, they encourage us to look again. They… change our mind.

Now, let's talk aesthetics.
The aesthetical result a mathematician feels after, through an exhausting line of work, manages to prove some mathematical theorem, is the same that a poet feels when they manage to perfectly express and display their ideas through a poem.

Both are products of fantasy, inspiration and talent. Besides, Poincare stated, among other things:
«A mathematician receives, from their work, the same sentiment as an artist does. Their joy is of the same quality and quantity.»

Paul Dirac, one of the founders of quantum physics, emphasized:

«We cannot determine mathematical beauty more than we can determine artistic beauty, but those studying mathematics have no trouble appreciating it.»
According to studies, a mathematical proof can stimulate the same part of the brain that is affected by art and our perception of beauty.
Poetry is quite beautiful, through its use of words and its meaning, we can all see that. Beauty in mathematics, though, comes in two different types; logic and visual. Logical beauty is described as cold and strict, but those who don't understand it. Visual beauty, on the other hand, is directly appealing, because it is symmetrical.
But what makes symmetry so engaging? The answer is repetition. Our brains like repetition, up to a point. A snowflake is pretty because it repeats the same shapes in symmetry. Music is enjoyable because, in its core, it is a rhythmically repeating sound. Of course, there are several themes and variations in music, but they, too, consist of repetitions.4

If we removed fantasy and inspiration from mathematics, we would be left with plain, appalling formulas, the ones our infertile, unimaginative educational system teaches. The same way, if we removed fantasy and inspiration from poetry, there would be only words, without meaning.

To summarize:
I hope that you can all see now how close, and ultimately relative, poetry and mathematics are to each other. How one completes and complements the other. How they shift our view of the world. And of course, how important it is for us to learn to use them.

THE PRINGLES® FUNCTION

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ABSTRACT

Crisps are a beloved snack in most households. What most households are unaware of, however, is that the shape of the iconic Pringles® crisps can be described quite simply using a mathematical function. This function is the hyperbolic paraboloid, $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = c \cdot z\ ,\ a, b, c \neq 0$. The hyperbolic paraboloid is a doubly ruled surface, consisting of infinitely many skew lines. For a Pringles® crisp, which exists physically and therefore occupies a finite space, a second condition must be also fulfilled; that of the super-ellipse, $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1$. The experimental values for the constants are $a = 2{,}25, b = 3{,}12,\ n = 4$, when, for simplicity, it is assumed that $c = 1$. The hyperbolic paraboloid is found in a number of real-world contexts, such as architecture and origami.

THE HYPERBOLIC PARABOLOID

The hyperbolic paraboloid is a three-dimensional geometric surface. The function for it is $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = c \cdot z\ ,\ a, b, c \neq 0$. For $a = b = c = 1$, or for what might be termed a "unit hyperbolic paraboloid" $x^2 - y^2 = z$, a plot is given in figure 1. The name comes from the parabolic cross-sections to be found when slicing the surface along the plane $y = x$, as well as the hyperbolas to be found in the contour plot for the surface.

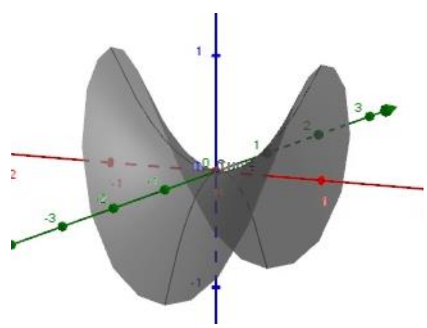

Figure 1

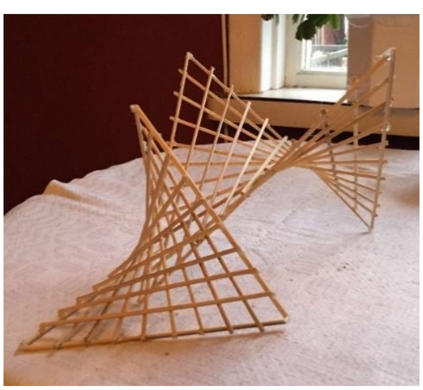

Figure 2

A model of the shape can easily be constructed by hand (see figure 2) as the hyperbolic paraboloid consists of two groups of skew lines. Skew lines are lines that are neither parallel or intersecting. Furthermore, the surface of the hyperbolic paraboloid is doubly ruled. This means that for any arbitrary point P on its surface, two distinct lines can be drawn that both lie on the surface.

The Gaussian curvature of a surface indicates how bent a surface is at a chosen point. This is done by multiplying the two principal curvatures of the point. These, in turn, are given by the curvature of two normal sections, the curves

formed where two normal planes intersect with the surface. The normal planes are the two planes where a normal vector to the chosen point can be found.
For a hyperbolic parabola, the Gaussian curvature is always lesser than zero. For values, except ones surrounding the origin point, the curvature is very close to zero.

THE PRINGLES® FUNCTION

The science of making Pringles® is chemistry, an inexact science. From this follows that the shape of Pringles® crisps is inexact as well. The constants a and b in the general formula $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = c \cdot z$ can be determined through physical measurements of the crisps. For simplicity, $c = 1$ is assumed to be true.
The hyperbolic paraboloid is defined for all values of x, y, and z. A single crisp has physical limitations. Therefore, the Pringles® function must be limited. Since the value of z is determined by x and y, a two-dimensional limitation of the last two works well. The x- and y-axes together form a plane in which, when observed straight-on, we see the hyperbolic paraboloid, or the crisp, from "above". A Pringles® crisp from above can be described as looking vaguely elliptical. However, the ellipse, $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$, is a faulty approximation, and the closest shape is a super ellipse. Super ellipses are described by $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1, \text{where } n \in Z^+$. When $n = 2$, the function describes a regular ellipse. The n that generates a surface closest to that of a Pringles® crisp seen from above is $n = 4$. This can be shown visually, as due to symmetry, a Pringles® crisp needs to have an even value of n. $n = 2$ very clearly is too round, while $n = 6$ is too square, leaving $n = 4$ as the only viable n.
The constants a & b are what determines an ellipse's semi-major and semi-minor axes. The semi-major and -minor axes are the points of an ellipse that are the furthest away from and closest to the origin point for an ellipse centred at the origin point. (This is also true for all super ellipses given that $n \geq 2$.) These points are located at the intersections between the ellipse and the coordinate axes, where either x or y equals 0. (This is true for super ellipses with any n.) For both a regular ellipse and a super ellipse this means that a & b are given by

$$\left.\begin{matrix}\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \\ y = 0\end{matrix}\right\} \rightarrow \left(\frac{x}{a}\right)^n = 1 \rightarrow \frac{x}{a} = \sqrt[n]{1} = 1 \rightarrow x = a$$

The same reasoning for $x = 0$ gives us that $y = b$ gives the other semi-minor or -major axis.
Since the making of Pringles® is a corporate secret, the easiest way to determine a & b is to measure the crisps at their widest width and length. This was done using a ruler and a measuring tape, both standard issue as found in most miscellaneous stores. Several crisps were measured. The averages of all measurements for length and width were taken separately, and the results were halved, since the measured values corresponded to the entire length and width, the distances between the two semi-major and -minor axes points, respectively. The centre of the Pringles® function is set in the origin point.
The result of the measuring process was that $a = 2{,}25$ and $b = 3{,}12$. (All in centimetres.) Since the hyperbolic paraboloid is limited by the super ellipse, it follows naturally that the "semi-minor and -major axes" of the hyperbolic paraboloid, as well, are equal to 2,25 and 3,12, respectively. To summarize, the Pringles® function is

$$\begin{cases} \left(\frac{x}{2,25}\right)^2 - \left(\frac{y}{3,12}\right)^2 = z \\ \left(\frac{\mathrm{x}}{2,25}\right)^4 + \left(\frac{\mathrm{y}}{3,12}\right)^4 = 1 \end{cases}$$

WHY IT WORKS AND OTHER USES

A Pringles® crisp has the shape it does for the following reasons: they are easy to stack, fit easily in a tube, break less, and the unique shape make them easily identifiable. Since the two points of either the semi-minor or the -major axes are touching whatever surface they are lying on, two points of balance are created. Their uniformity means that the entire surface of one crisp touches the entire surface of another crisp. There is very little room for “wiggling”, which is what makes them easier to stack, especially in a tube as they have a semi-round shape, and less likely to break in their tube.

Aside from its place in pure mathematics, the hyperbolic paraboloid can be found in varying contexts. As it is a fascinating shape to look at, it is often used in architecture, such as the Ochota railway station roof in Warsaw, Poland. The negative Gaussian curvature means that rain and other downpour easily slides off a structure made up of a hyperbolic paraboloid, which is useful for roofs. Approximations of the surface serve as playground equipment, and a model of the hyperbolic paraboloid can be folded out of simple paper.

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MAGIC SQUARES

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ABSTRACT

A magic square is a square grid filled with numbers, in such a way that each row, each column, and the two diagonals add up to the same number. The n × n magic square has n rows and n columns, and two diagonal. If you add up the numbers in any row, column or diagonal, you always get a constant number.

The squares that have n by n are home and the sum of the numbers of rows, columns, and diagonals is constant. Also, for arbitrary numbers, a square can be constructed, which means that the numbers are in the table so that the sum of rows, columns, and diagonals is the arbitrary number. In this article we successfully demonstrate, first, how to generate the base magic squares, and then how to generate a magic square for arbitrary numbers.

GENERATING 3 BY 3 MAGIC SQUARES

A magic square is a matrix filled with numbers, in such a way that each row, each column, and the two diagonals add up to the same number. The 3 × 3 magic square has three rows and three columns, and if you add up the numbers in any row, column or diagonal, you always get a constant number.

The base 3 × 3 magic square has 9 houses, in which houses the numbers from 1 to 9 are filled in such a way that the sum of rows and columns is a constant number. In 3 × 3 magic square, magic number is 15. This number is obtained by dividing the total number of table numbers by the number of rows or columns.

$$S = \sum_{1}^{n} n = 1 + 2 + 3 + \cdots + 9 = \frac{n(n+1)}{2} = 45$$

$$A = \frac{\sum_1^n n}{R} = \frac{45}{3} = 15$$

Here is a 3 × 3 magic square

4	9	2
3	5	7
8	1	6

The central number of 3 × 3 magic square is 5. Table homes can be filled in different modes. In filling the tables in the table, two conditions must be met. The central number of the table should be 5 and the sum of the numbers should be 15.

Formula of generating 3 × 3 magic square: The following technique is one of the easiest ways to generate 3 × 3 magic square:

n+3	n-4	n+1
n-2	n	n+2
n-1	n+4	n-3

By placing n = 5, the following table is obtained:

8	1	6
3	5	7
4	9	2

Multiply the numbers of a magic square in a constant number. If we multiply the numbers of a magic square in a constant number C and accumulate with a constant x, the resulting square will also be magic square. Therefore, an extremely magic square can be created.

8a+b	1a+b	6a+b
3a+b	5a+b	7a+b
4a+b	9a+b	2a+b

Adding constant Numbers to magic square: By adding a constant number to all magic squares, the resulting square will also be magic square.
Consider the square of 3 on 3 bases. If all the houses have a constant number, the square will be successful.

8	1	6
3	5	7
4	9	2

By adding a constant number like 2 to all the houses in the base magic square is as follows:

10	3	8
5	7	9
6	11	4

If a constant number such as 9 is added to all its houses, the resulting square will be a magic square. as a result The sum of the layers and columns and the diagonal is a constant number.

17	10	15
12	14	16
13	18	11

8	1	6
3	5	7
4	9	2

By adding a constant number of 1, 2, 3, and 4 and using one of the 8 magic square pattern below can be infinitely generated magic squares.

3	10	5
8	6	4
7	2	9

6	11	4
5	7	9
10	3	8

7	6	11
12	8	4
5	10	9

10	5	12
11	9	7
6	13	8

If we multiply all the numbers in 3 × 3 magic square in 2 and then add the constant number 9, we see that the sum of rows and columns is 57. Therefore, the resulting square is magic.

25	11	21
15	19	23
17	27	13

If we multiply all the numbers in 3 × 3 magic square in 2 and then add the constant number 9, we see that the sum of rows and columns is 57. Therefore, the resulting square is magic.

GENERATING 4 BY 4 MAGIC SQUARES

The base 4 × 4 magic square has 16 houses, in which houses the numbers from 1 to 16 are filled in such a way that the sum of rows and columns is a constant number. In 4 × 4 magic square, magic number is 34. This number is obtained by dividing the total number of table numbers by the number of rows or columns.
Total numbers of table houses is:

$$S = \sum_{1}^{n} \mathrm{n} = 1 + 2 + 3 + \cdots + 16 = 136$$

The magic number is:

$$\mathrm{A} = \frac{\sum_1^n \mathrm{n}}{\mathrm{R}} = \frac{136}{4} = 34$$

Here is a base 4 × 4 magic square:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

The following technique is one of the easiest ways to produce 4x4 square squares. We shift the numbers on the square diameters symmetrically. We replace positions 1 and 16 with each other and replace the numbers 6 and 11 with each other. We replace 4th, 13th, and 7th and 10th places together.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

We see that the total number of the table is 136 and the sum of the rows and columns is 34.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

GENERATING 5 BY 5 MAGIC SQUARES

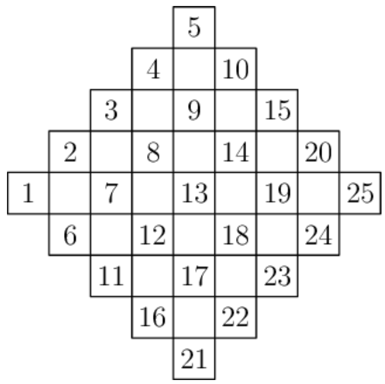
The base 5 × 5 magic square has 25 houses, in which houses the numbers from 1 to 25 are filled in such a way that the sum of rows and columns is a constant number. In 5 × 5 magic square, magic number is 65. This number is obtained by dividing the total number of table numbers by the number of rows or columns.

Total numbers of table houses is:

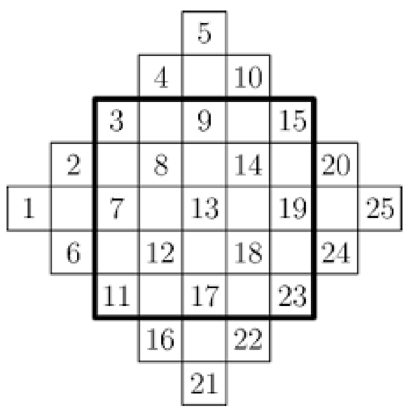
$$S = \sum_{1}^{n} \mathrm{n} = 1 + 2 + 3 + \cdots + 16 = 325$$

$$\mathrm{A} = \frac{\sum_{1}^{n} \mathrm{n}}{\mathrm{R}} = \frac{325}{5} = 65$$

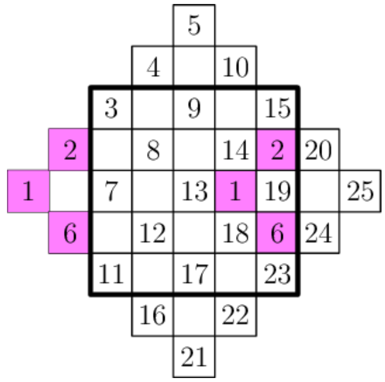
A technique for generating 5 × 5 magic square: To create a 5 × 5 magic square, first we create the following shape and fill numbers 1 through 25 as follows:



We will denote the square of 5 in 5 in the form and keep the numbers inside the square constant and do not change.



Move each of the numbers outside the square of 5 in 5 to the last empty house in the same row or column.



So the square of the base 5 to 5 square is created as follows. The sum of numbers in this table is 325, and the sum of rows, columns, and diameters is 65.

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

There are several models of 5 to 5 magic squares. One of them is:

7	13	19	25	1
20	21	2	8	14
3	9	15	16	22
11	17	23	4	10
24	5	6	12	18

Create a magic square for each arbitrary number: The magic square is a matrix n in n, which has n^2 houses, in which the numbers from 1 to n^2 are filled in such that the sum of rows and columns is a constant number. In 3 × 3 magic square, magic number is 15. This number is obtained by dividing the total number of table numbers by the number of rows or table columns. The magic number for a 4 × 4 magic square is 34 and for a 5 × 5 magic square is 65, and for a 6 × 6 magic square is 111, and for a 7 × 7 magic square is 175, and for a 8 × 8 magic square is 260, and for a 9 × 9 magic square is 369 and for 10 × 10 magic square is 505. Therefore, for numbers less than 15, you cannot create a 3 × 3 magic square, and you cannot create a 4 × 4 magic square for numbers less than 34, so you cannot create a 10 × 10 magic square for numbers less than 505.

GENERATING A 3 × 3 MAGIC SQUARE FOR EACH ARBITRARY NUMBER

This Square has 9 houses. The order of filling numbers is very important. In this square, the sum of rows and columns is 15. For numbers less than 15 you cannot make a square. There are two modes for building a 3 × 3 magic square.

A. The arbitrary number after reduction divided by 3 and division has not remainder: you cannot make a square. There are two modes for building a 3 × 3 magic square.
For example, for creating a 3 × 3 magic square for number 72 as follows:72 subtract 12 is 60 and this number divided by 3 and division has not remainder. If we divide 60 into 3 the result is 20 and we have not reminder. Therefore 20 is start number. We put 20 in the first house. Then we add the number 1 to the next house. So we fill to the last house. Due to the fact that the result of the division is not left, it does not need to be corrected.
Reduction number $72 - 12 = 60$
Start number $60 \div 3 = 20$
We will consider the base 3 × 3 magic square:

4	9	2
3	5	7
8	1	6

The corresponding 3 × 3 magic square for 72 is:

23	28	21
22	24	26
27	20	25

B. The arbitrary number after reduction is not divided by 3 and division has remainder: In this mode for creating a 3 × 3 magic square for arbitrary number as follows: In this mode for creating a 3 × 3 magic square for arbitrary number as follows: We subtract arbitrary number 12. Then we divide the result to 3. If the result of division has remainder, we put start number in the first house. Then we add the number 1 to the next house. So we fill to the last house and we will do According to the table below.

How to fill the table	reminder
We put start number in the first house. Then we add the number 1 to the next house. So we fill to the last house.	0
We put start number in the first house. Then we add the number 1 to the next house. We add one at house 7. Then fill the next houses naturally.	1
We put start number in the first house. Then we add the number 1 to the next house. We add one at house 5. Then fill the next houses naturally.	2

Remember that 3 × 3 magic square has 3 rounds. First round: 1-2-3, Second round: 4-5-6, third round: 7-6-8. The houses for change to create magic squares:

4	9	2
3	5	7
8	1	6

For example the 3 × 3 magic square of 25 is created as follows: 25 subtract 12 is 13 and this number divided by 3 and division has a remainder. If we divide 13 into 3 the result is 4 and we have one reminder. Therefore 4 is start number. We put 4 in the first house. Then we add the number 1 to the next house. So we fill to the last house.
Reduction number $25 - 12 = 13$
Start number $13 \div 3 = 4$
And it has one reminder
We will consider the base 3 × 3 magic square:

4	9	2
3	5	7
8	1	6

And it has one reminder. We put start number in the first house. We add one at house 7. Then fill the next houses naturally.

7	13	5
6	8	11
12	4	9

For creating the n × n magic square for arbitrary number, we act like 3 by 3 magic square. It should be noted that the reduction number for 3 by 3 magic square is 12 and for 4 by 4 magic square is 30 and for 5 by 5 magic square is 60.

CONCLUSION

For arbitrary numbers, a magic square can be constructed. For creating the n × n magic square for arbitrary number, we subtract reduction number from arbitrary number. The reduction number for 3 by 3 magic square is 12 and for 4 by 4 magic square is 30 and for 5 by 5 magic square is 60. Then we divide the result to n. If the result of division has not remainder, we put start number in the first house. Then we add the number 1 to the next house. So we fill to the last house. And if the result of division has a remainder we add number 1 in the one of the houses $3n + 1$, $4n + 1$, $5n + 1$ number one , and In the remaining houses we add naturally number one unit.

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SEQUENCE RESULTING AS SOLUTION OF TWO COMBINATORIAL PROBLEMS

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ABSTRACT

Often combinatorial problems are very difficult. However, a difficult combinatorial problem can be simplified that is the goal of science – to serve people. For example, a common problem: a city has the form of a rectangle and a network of n streets towards North-South and m streets towards East-West. How many ways are there for tourist to get from vertex South-West to the North-East one of the city moving in the direction from West to East and from South to North across the streets. This problem can be reduced down to the similar one that is about the number of ways to place m red balls and n yellow ones in a tube. Our purpose is to examine the sequence resulting as the solution of 2 combinatorial problems. First problem: $2 \times k$ an $2 \times n$ rectangles are given; dominos represent 1×2 rectangles. How many ways are there to full $2 \times k$ and $2 \times n$ rectangles with dominos so that they will contain an equal number of vertical dominos? Second problem: two parallel lines are given. The first contains k, the second one n indicated points. These points form pairs so, that segments, connecting points in each pair, do not intersect. What is the number of pairs? Solutions of these two problems result in a reasonable sequence. In our work we study properties of this sequence and its connection with other well-known sequences.

Introduction

In this work we regard, how identic sequences can appear in different combinatorial problems. This research was inspired by the following problem, which was offered on Saint-Petersburg Math Olympiad in year 2014:

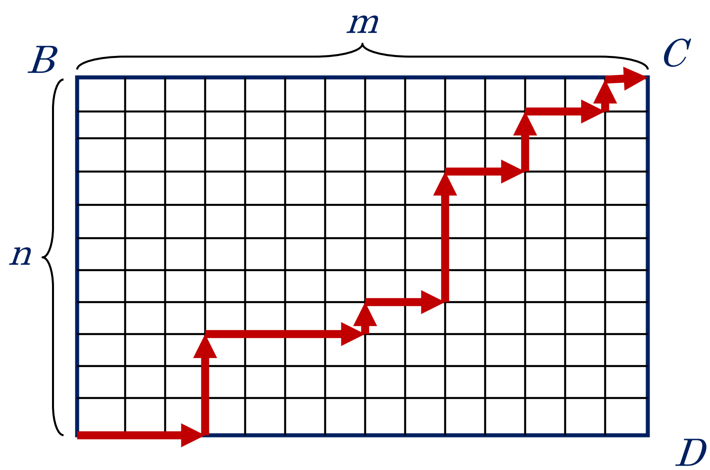
"On each of two parallel lines 40 points are indicated. They are connected into pairs so that segments, which connect points in each pair do not intersect (in particular, no point belongs to several segments and no point does not have a pair). Prove that the number of ways to form such configurations is less than 3^{39}"

So, we will make generalization of this combinatorial sequence. In order to make clearer the context of this work, in part I we will represent on the example of another three combinatorial problems, how different problems can be reduced to common sequence and common solution.

I

"Town m, n" problem

A town in form of a rectangle is given with vertexes: *A*(south-west), *B*(north-west), *C*(north-east), *D*(south-east). The streets are situated parallel to AB or parallel to *BC*. Let *n* – be length of *AB*, *m* – length of *BC*. The tourist sets from *A* to *B*, passing the streets of the town so that in the northern or eastern direction. How many ways are there for the tourist to manage that?



Following the data of the problem, *n* – the distance passed in the northern direction, *m* – the distance passed in the eastern direction. Let $l_1, l_2, l_3, \ldots, l_m$ - be the fragments of the northern way and $k_1, k_2, k_3, \ldots, k_n$ - be the fragments of the eastern way. Then number of possible ways for the tourist to reach the point *A* from point *B* can be found by solving in positive integers equation:

$$l_1+ l_2+ l_3+ \ldots+l_m =n$$

or equation:

$$k_1+ k_2+ k_3+ \ldots+k_n = m$$

Obviously, the choice of northern way determines the choice of the eastern ways (therefore, chose of the whole way) and vice versa. So, to answer the question of the problem is enough to solve one of equations, demonstrated above.

And on this point the given problem intersects with well-known **Moivre problem**

"How many positive integer solutions does the equation $x_1+ x_2+ x_3+ \ldots+x_n = k$ *have?"*

On this step, not solving these two problems, we offer to regard another combinatorial problem, which will help to solve last ones.

Tube problem

A tube is given. It is filled with blue and red balls of the same size (in particular, radius of the bottom equals radius of the balls, so that balls can be placed in the tube one by one in vertical trajectory) in the following way: first, k_1 blue balls are placed, the one red one is added, after that k_2 blue balls are added and the one red ball is added and so on, …, finally, k_n blue balls are added and the last red ball is added. So, n – is the number of red balls,
$k_1+ k_2+ k_3+ \ldots+k_n = m$ – the number of blue balls.

How many ways are there to place *m* blue balls in tube?

To manage that we need to solve in positive integers equation $k_1+ k_2+ k_3+ \ldots+k_n = m$ – here we see identity with Moivre problem (theory of numbers) and identity with "town *m*, *n*" problem.

1) How many ways are there to situate *n-1* red balls in the tube? (we do not take into account one red ball, because it is always on the top of the tube)

Taking into consideration that there are *n-1* red balls and *m+n-1* all balls, we can answer question 2), using the combinatorial formula:

$$C_{n+m-1}^{n-1}$$

and using a well-known combinatorial property we can transform this formula in such a way:

$$C_{n+m-1}^{n-1} = C_{n+m-1}^{m+m-1-(n-1)} = C_{n+m-1}^{m}$$

So, we got formula for calculating number of ways to situate *m* blue balls, which, as it is proved above, equals the number of ways to situate *n* red balls.

We easily see that Moivre problem and tube problem are solved using the same formula:

$$C_{n+m-1}^{n-1} = C_{n+m-1}^{m+m-1-(n-1)} = C_{n+m-1}^{m}$$

And, as Moivre problem and "town *m*, *n*" problem are identic, this formula solves all the three problems.

So, we got the following scheme: while solving "town *m*, *n*" problem we noticed that it is identic to Moivre problem. With the help of identic sequence from tube problem we got the common solution and the common sequence.

II
"Configurations

Now let's return to Saint-Petersburg Olympiad problem, making the generalization of this problem.

Two parallel lines are given. On the first line *k* points are indicated, on the second one *n* points are indicated. These points are connected into pairs so that segments, which connect points in each pair do not intersect (in particular, no point belongs to several segments and no point does not have a pair, which means that *k* and *n* are of the same parity).

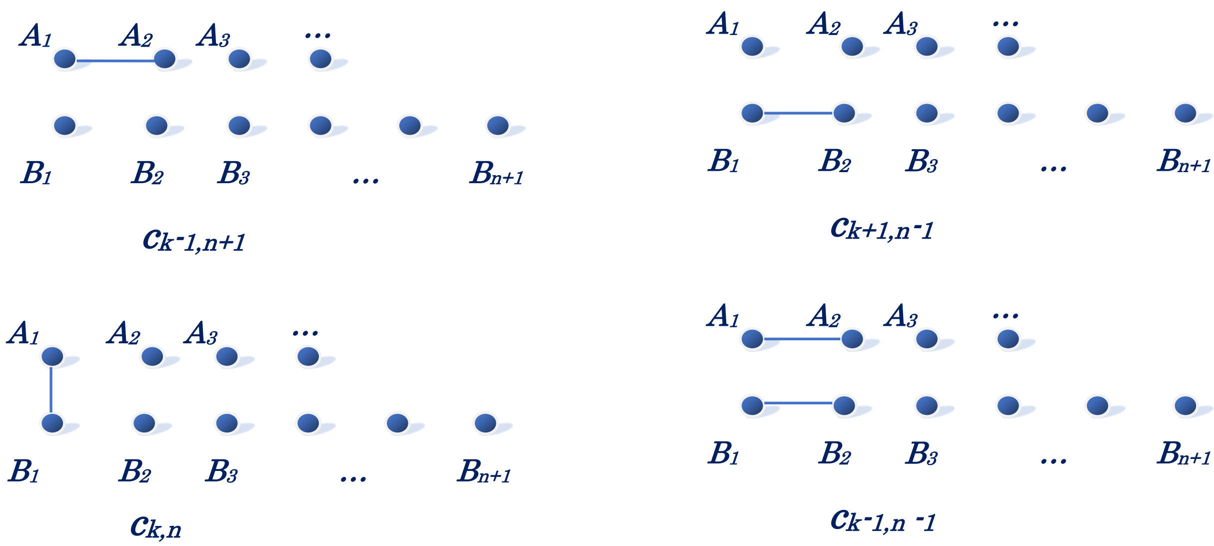
Let such combinatorial constructions be named "configurations" and the number of all possible "configurations" be $c_{k,n}$.

If $k=2$ and $n=4$, then “configurations” will look in the following way:

As one can see on the pictures from above $c_{2,4}=4$.

Now let’s represent the number $c_{k+1,n+1}$ in the recurrent form. Let on the first line be $k+1$ points, on the second line $n+1$ points; let $A_1, A_2, A_3, \ldots, A_{k+1}$ – be the points indicated on line 1, $B_1, B_2, B_3, \ldots, B_{n+1}$ – be points indicated on line 2. Thus, the number of “configurations”, which contain the segment A_1A_2 is $c_{k-1,n+1}$; the number of “configurations”, which contain segment B_1B_2 is $c_{k+1,n-1}$; the number of configurations, which contain segment A_1B_2 is $c_{k,n}$. And now, as we would like to represent the number $c_{k+1,n+1}$ through numbers $c_{k-1,n+1}$, $c_{k+1,n-1}$, $c_{k,n}$, we notice that we twice take into account configurations, which contain both segments A_1A_2 and B_1B_2, and the number of such “configurations” is $c_{k-1,n-1}$.

Here graphically is represented the picture of “configurations”:

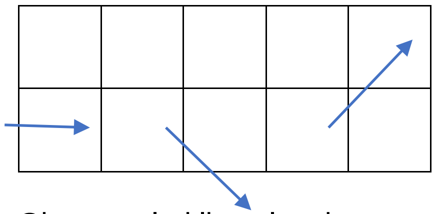


Thus, we can represent $c_{k+1,n+1}$ through numbers $c_{k-1,n+1}$, $c_{k+1,n-1}$, $c_{k,n}$, getting the following recurrent sequence:

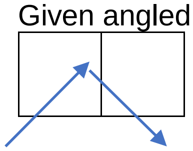
$$c_{k+1,n+1} = c_{k-1,n+1} + c_{k+1,n-1} + c_{k,n} \quad (1)$$

Motskin's "no-pick" routes

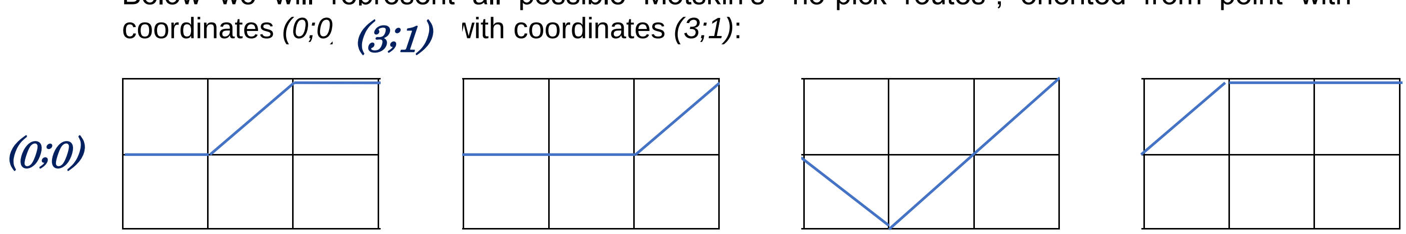
Motskin's "no-pick" route is an oriented angled line, situated on coordinate plane, each fragment of which is situated in one of the directions represented below:



Given angled line also does not have any "picks", the fragments of the following kind:



Below we will represent all possible Motskin's "no-pick routes", oriented from point with coordinates *(0;0* vith coordinates *(3;1)*:



Let the number of all possible Motskin's "no-pick routes", coming from point *A(0;0)* into point *Z(k;n)* be $b_{k,n}$.

Now, let's see how Motskin's "no-pick" route can reach the point *Z(k;n)*. Clearly, from point *X(k-1,n)* or from point *Y(k-1;n-1)* or from point *V(k-1;n+1)*; but it cannot reach the point *V(k-1;n+1)* from the point *P(k-2;n)*, because in such a case in the point *V(k-1;n+1)* a "pick" appears.

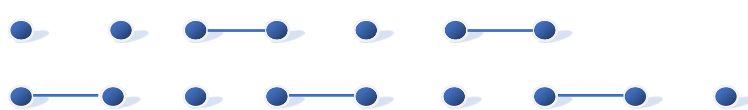
Thus, we get recurrent sequence in the following way:

$$b_{k,n} = b_{k-1,n} + b_{k-1,n-1} - b_{k-2,n} \quad (2)$$

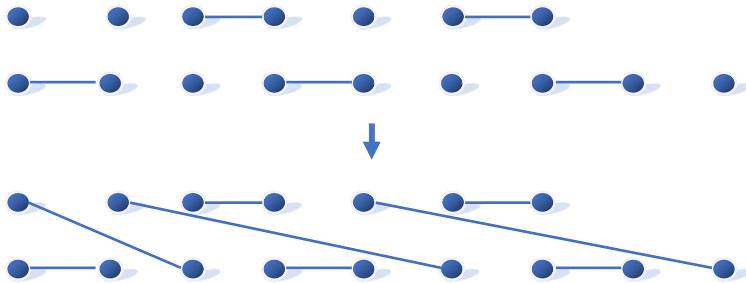
It is not difficult to notice that sequence 1 and sequence 2 are almost the same and are absolutely identic in structure and principle, which proves connection between combinatorial construction "configurations" and properties of Motskin's "no-pick routes", to which, as we have demonstrated, combinatory can be applied and which are related to the theory of graphs.

Calculating $c_{k,n}$

Let's return to "configurations" and calculate $c_{k,n}$. After we arrange "configurations", we erase segments, which connect into pairs points, which lay on different lines. So, from all the segments we leave only the horizontal ones:



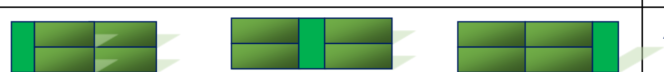
We notice that using the set of horizontal segments we can reestablish the initial picture:



So, we conclude that the choice of horizontal segments determines the choice of other segments and, thus, the picture of “configurations”, which means that, if we find the number of ways to choose horizontal segments, we automatically find $c_{k,n}$ as well.
On this point we temporary leave target to find $c_{k,n}$ and regard another combinatorial construction.

Domino

Domino – is a rectangle of the kind *1×2* or *2×1*. Let $d_{k,n}$ – be the number of ways to place in rectangle of the size *2×k* and *2×n* vertical dominos, and *j* – the number of vertical dominos in rectangle. Below we represent this in a table:

k	*n*	*j*	*2×k*	*2×n*	*d*	$d_{k,n}$
3	*5*	*1*			*4*	*10*
		3			*6*	

Now we set target to find out how we can calculate the number of ways to place vertical dominos in rectangles, because the number of these ways, obviously, determines the whole picture of filling rectangles with dominos.
But first, to find out that, we will regard an auxiliary problem, which will help us to find $c_{k,n}$ as well.

“Configurations *j, l* ”

So, auxiliary problem: let “configurations *j*, *l* ” on a line be a set, which consists of *l* not intersecting segments and *j* points, which do not lay on these segments. Let the ends of the segments be green points, other points be red. How many “configurations *j*, *l* ” on a line are possible?
To get that picture, let’s, firstly, draw *l* green points and *j* red points, then on the right side of each green point add one more green and connect green points with segments into pairs. During this operation we can notice that the choice of the number of initial green points determines the picture of “configurations *j*, *l* ”. And the number of ways to choose green points can be found by formula C_{j+l}

l – is the number of initial green points and $j+l$ – is the initial number of all points.
Using a well-known combinatorial property and representing the final number of points as $k=j+l$ (k and j are of the same parity), that formula can be got in such a way:

$$C_{j+l}^{l}=C_{\frac{k+j}{2}}^{\frac{k-j}{2}}=C_{\frac{k+j}{2}}^{\frac{k+j-(k-j)}{2}}=C_{\frac{k+j}{2}}^{j}$$

We can see that the number of ways to choose red points also determines the picture of "configurations j, l ".

Provement: $c_{k,n}= d_{k,n}$

And now we notice the connection between domino and "configurations j, l ": red points correspond to vertical dominos and pairs of green points connected with segments correspond to horizontal dominos.
So, we can apply the last formula for "configurations" on two lines and finally calculate $c_{k,n}$:

$$c_{k,n}=\sum_{\substack{0\le j\le \min\{k;n\}\\ k\equiv n\ (\mathrm{mod}\ 2)}} C_{\frac{k+j}{2}}^{\frac{k-j}{2}} C_{\frac{n+j}{2}}^{\frac{n-j}{2}}=\sum_{\substack{0\le j\le \min\{k;n\}\\ k\equiv n\ (\mathrm{mod}\ 2)}} C_{\frac{k+j}{2}}^{j} C_{\frac{n+j}{2}}^{j}$$

And, as we have demonstrated parallelism between domino and "configurations", we can represent identic formula for domino in rectangles $2\times k$ and $2\times n$:

$$d_{k,n}=c_{k,n}=\sum_{\substack{0\le j\le \min\{k;n\}\\ k\equiv n\ (\mathrm{mod}\ 2)}} C_{\frac{k+j}{2}}^{\frac{k-j}{2}} C_{\frac{n+j}{2}}^{\frac{n-j}{2}}=\sum_{\substack{0\le j\le \min\{k;n\}\\ k\equiv n\ (\mathrm{mod}\ 2)}} C_{\frac{k+j}{2}}^{j} C_{\frac{n+j}{2}}^{j}$$

Conclusion

So, in part II we have generalized the sequence of the problem offered on Saint-Petersburg school Olympiad. Like in part I we have demonstrated how in particular combinatorial constructions the choice of objects of one kind determines the choice of objects of another kind; how common properties and sequences appear in different mathematical constructions. And in the whole work we have connected different fields of mathematics: combinatory, theory of numbers (Moivre problem) and theory of graphs (Motskin's "no-pick" routes).

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A CIRCLE ROTATES TOWARDS EXCITING MATHEMATICS

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ABSTRACT

Suppose you wish to construct a waterslide that will take you in the least time down to a given point. Which would the waterslide's curve be?
Also, imagine that you have a circle rotating inside another circle of twice the diameter. What shape would any point on the small circle make?
In our paper, we address these two problems that have one main common characteristic: the (rotating) circle. We also show that these problems are not only brainteasers with mind-blowing solutions, but, more importantly, the Brachistochrone led to the rise of fields within Mathematics such as the Calculus of Variations.

INTRODUCTION

"The circle" is an object, a drawing, a geometrical shape that everyone recognises. It is used to describe cyclic forms for a variety of things in our everyday life. In this paper, we examine two applications of the circle, namely the Brachistochrone and the Tusi Couple.

The Brachistochrone problem: Imagine that you have two points in a vertical plane and you want to go from point A to point B travelling only by the force of gravity in the fastest way possible. How does the route that you should follow look like? Johann Bernoulli first addressed this problem to the readers of Acta Eruditorum in 1696 [5]. Some ideas would be the straight line, the arc of a circle, the semi-ellipse, the cycloid, an extreme curve, or any other curve one may imagine. In fact, it was proven in 1697 that the cycloid gives the fastest route [5].
What is more fascinating is that the cycloid curve is the only curve that possesses the Tautochrone property: an object will reach the bottom (point B) at the same amount of time independently of its starting position on the curve!

The Tusi Couple: A small circle C1 of a given diameter constantly rolls inside a bigger circle C2 of double the diameter. If a random point M that belongs to the circumference of C1 had the ability to draw its route as it rolled inside the other circle, what shape would it make?
The solution to this brainteaser is mind-blowing and we provide the proof in the respective section.

THE BRACHISTOCHRONE PROBLEM

The problem of finding the fastest way to go from one place to another is one that we face in our daily life. In Mathematics, we have various ways to find solutions to such problems, such as

Graph Theory. However, in this paper we focus on the problem of the fastest route on a vertical plane, on which an object slides frictionless to a given end-point under only gravity.

A simple everyday task or a brainteaser may have been what inspired Galileo in 1638 to find which exact route an object should follow in order to reach the bottom of the curve in the least time.
The name Brachistochrone comes from the Greek words brachis, which means shortest, and chronos, which means time. Many prominent mathematicians of that time tried to solve or provided a solution to this problem, including Newton, Leibniz and l'Hôpital as well as Johan Bernoulli and his brother Jakob. Each of them found that the curve that solves the brachistochrone problem is actually the cycloid [2,3,5].

Figure 1: The construction of a waterslide between two points.

Imagine the construction of a waterslide that would take you from point A to point B as in Figure 1. How should one connect these two points in order to take you from point A to point B in the least time?

Obviously, there is an infinity of options. We suppose that we have the five options as in Figure 2.

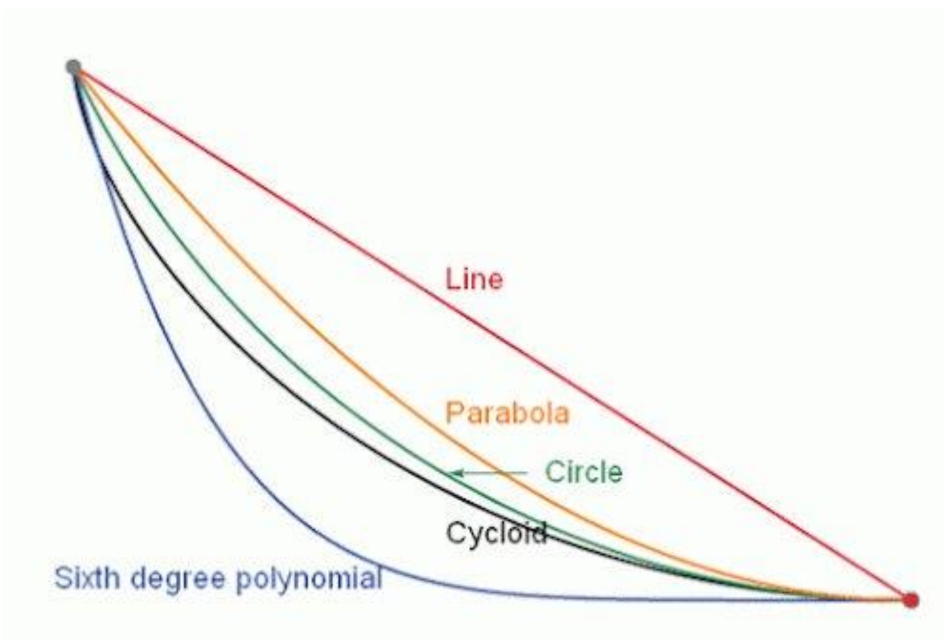


Figure 2: Five possible options for the Brachistochrone problem.

To begin, we should see if the simplest option we have, the straight line, is the answer to our question.
The straight line surely is the shortest path between the two points but speed increases at a constant rate on a straight line. As a result, we need to find a way to give our object more acceleration.

The extreme curve (say a sixth degree polynomial) can give our object the maximum acceleration, but the distance that it will need to cover is also maximized.

So the answer should lie somewhere between the straight line and the extreme curve. In fact, the right curve for our problem is the cycloid. The cycloid manages to increase speed and minimize the distance in an optimal way. In order to explain why the cycloid is the answer to this problem, many mathematicians of the 17th century had to create a completely new field in Mathematics, called the Calculus of Variations [3].
Galileo mistakenly proposed the arc of a circle as the answer but later the cycloid was proven to be the solution by Bernoulli, using Snell's law.

In order to understand the thinking behind the solution of Bernoulli, we examine a simpler problem [4]: Suppose someone is standing on a surface covered with mud and he wants to reach a ball located on a street in the shortest time as in Figure 3. Which is the best route to follow?
The first thing that probably comes to mind is to run straight to the ball.

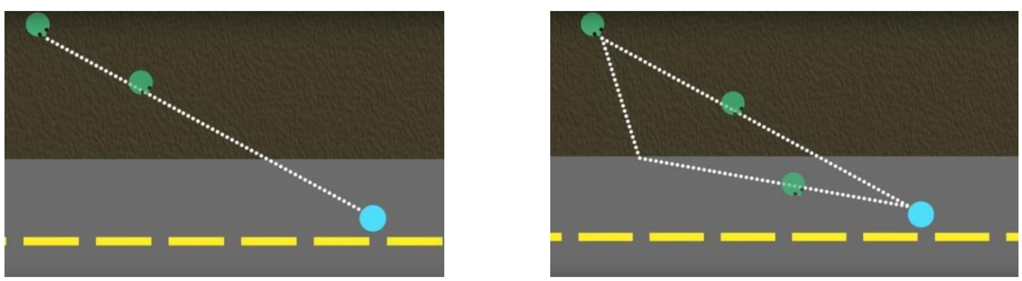
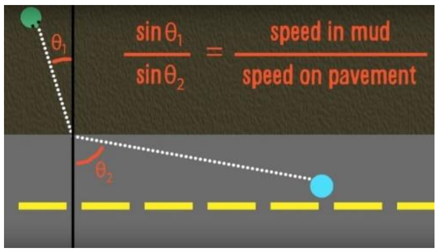


Figure 3: A problem that introduces Snell’s Law.

Notice that running on the street is faster than running on the mud. As a result, the straight line is not the quickest path. Thus, the best path to follow is the one where less distance is travelled on mud, which is the more challenging material to move on, and a longer one on the surface that someone moves easier on, the street. In this way, the actual speed is maximized.



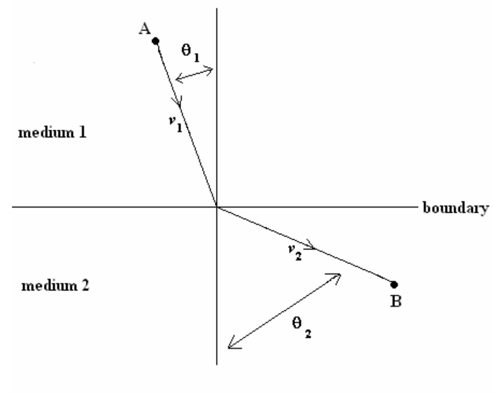


Figure 4: Snell's Law

So, as the person enters the street two angles are formed, θ1 and θ2.
If the ratio of sinθ1 and sinθ2 is equal to the ratio of the speed in mud and the speed on the street (see Figure 4), then the resulting line is the fastest route. In other words, Snell's Law, which is also known as the law of refraction, states that when a ray of light crosses the boundary between transparent media, it experiences a change in direction characterized by the relation [2,4]

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\upsilon_1}{\upsilon_2}$$

in which θ1 is the angle of incidence, θ2 is the angle of refraction and υ1 and υ2 are the velocities depending on the nature of the two media (Figure 4).

According to the principle of least time, light will always travel along a path with the shortest traversal time and apparently, what Bernoulli did was to add thinner and thinner layers for the light to move on. In fact, consider a situation with a light ray traveling downward through many transparent layers, with each medium less dense than the layer above it. The speed of light increases in every next media as it progresses through deeper layers and the ray of light bends further away from one to another medium at point of contact. As the number of layers increases to infinity, the path becomes a curve that is actually the cycloid [2,4].

CYCLOID

The cycloid is the curve traced by a point on a circle when the circle rotates on a straight line as in Figure 5.

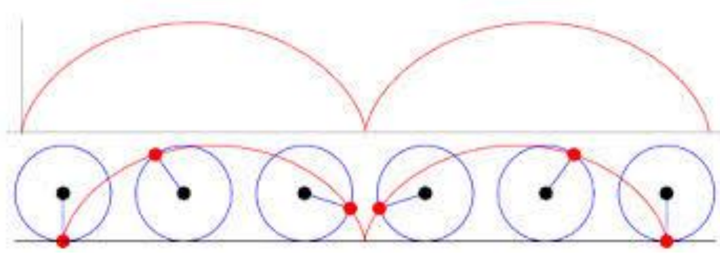


Figure 5: The cycloid.

We derive the parametric equations [1] of the cycloid because the Cartesian equation is hard to be identified and a closed-form expression of the form $y = f(x)$ is not possible [6].

In Figure 6, a circle was captured while moving, so we can analyze the cycloid while it is being created. The image is a snapshot of the same circle as it rotates on the x-axis and forms the cycloid.

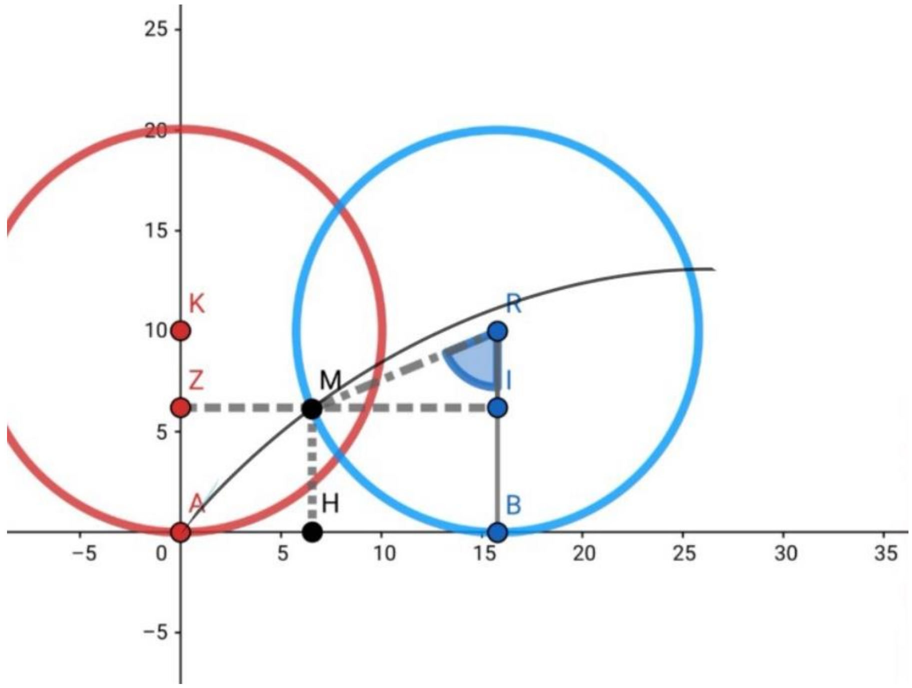


Figure 6: Proof of the parametric equations for the cycloid.

In Figure 6, K and R the circles' centers and KA and RB are their radii. The point M is where point A is found in the blue snapshot. Points I,Z and H are projections of point M on $RB,\ KA$ and the x-axis, respectively. The angle θ, which is the parameter of the equations, represents the angle of rotation of the circle, and in the blue snapshot is $\widehat{MRI}$.
So in order to find the parametric equations of the cycloid, we need to find the coordinates y and x of the random point M, in other words ZA and AH.

Starting with the y coordinate:
$y = ZA\ = AK - ZK\ = AK - IR$.
However, working on the right-angle triangle MRI
$IR = MR\ \cdot cos\theta = r \cdot cos\theta$.
Therefore $y = r \cdot (1 - cos\theta)$ since AK and MR are both radii of the circle.

Now for the x coordinate:
$x = AB - HB = AB - MI = AB - MR \cdot sin\theta$.
However, AB is equal to the length of the arc MB and $MB = \theta \cdot r$ (with θ measured in rad), therefore $x = r\theta - rsin\theta = r(\theta - sin\theta)$
Hence, the parametric equations of the cycloid are:

$x = r\ (t - sint)$
$y = r\ (1 - cost)$
where t is the angle through which the circle is rotated, the parameter, and this is the solution of the Brachistochrone problem.

Similar to the previously analysed barchistochrone is the tautochrone problem. The word derives from greek; tauto means “the same” and chrone means “time” and this is exactly what this problem is all about, that is: finding the curve for which the time taken by an object sliding (without friction) to its lowest point is independent of its starting point on the curve. The solution is, surprisingly, the cycloid and Huygens first published this result in 1673 during his work on developing a reliable pendulum clock. These clocks swing in cycloidal arcs because then the pendulum takes the same time to make a complete oscillation whether it swings through a wide or a small arc [2].

TUSI COUPLE

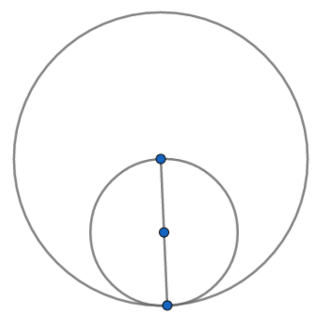


Figure 7: The Tusi Couple

The Tusi Couple is a relatively known mathematical problem, and this is what it consists of: Suppose there is a circle with a certain diameter, and inside of it, is another one with half the diameter and tangent on one point to the bigger one as in Figure 7. If the smaller circle starts rotating inside of the bigger one, without ever stopping being tangent to it, what shape will any point of the given small circle form?

The problem was first introduced and solved by the Persian astronomer and mathematician Nasir Al-Din Al-Tusi, in 1247 [7].

In fact, it is said that the famous astronomer Copernicus used it in multiple occasions such as the study of inferior planets [7]. However, its interest does not lie solely on its uses and its unexpected solution, but also on the unique way it needs to be approached in order for it to be solved. More precisely, in order for one to prove the solution to the problem, one needs to already know the answer, in contrast to the majority of the mathematical problems where one finds the answer whilst working on and exploring the given problem. That is why, Tusi, in order to prove the solution, first established the answer out of intuition and through experiments and then started working on the proof.

The answer was proved in 1259 to be a straight line. The proof surely is unexpected but, from a mathematical point of view is not complex. As previously mentioned, the complexity of the problem lies in finding the correct answer, not establishing it. For an animated model see [4,7].

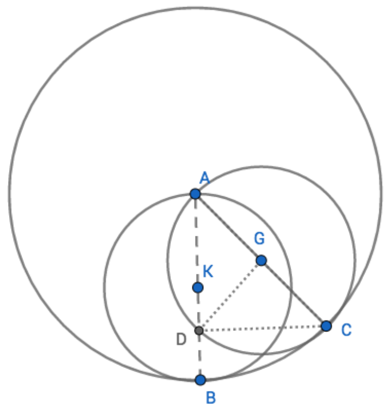


Figure 8: Proof of the Tusi Couple.

However, how can one mathematically provide the answer?

To start with, we have the circle $(K, KB = r)$ as in Figure 8, tangent to the circle $(A, AB = R)$ with $R = 2r$. If the smaller circle starts rotating, like in our problem, the point B will become another point. We name it D. To prove that the straight line is indeed the answer it suffices to prove that the points A, B and D are collinear.

Three basic ways enable one to prove that three points lie in the same line:

- Calculate the area of the triangle formed by these three points. If the area is equal to zero then the three points are part of the same line.
- Prove that two angles (formed by two of our points and an extra point, away from the other three) are equal. Say that the three points are A,B,C and the extra point is K. If we can prove that $\widehat{KAB} = \widehat{KAC}$ then we have proven that the three points are collinear.
- Use the mathematical equations of the straight line to find a line whose equation satisfies all three points.

In our case, the second option is more fitting. In result, to prove that the straight line is the answer it suffices to prove that the angle $\widehat{CAB} = \widehat{CAD}$.

First, we will look at the circle (A, AB) in which we have:
$\widehat{CAB}$ subtends to the arc BC and therefore $BC = \widehat{CAB} \cdot R = \widehat{CAB} \cdot 2r$

Similarly, in the circle (G, GC) we have:
$\widehat{CGD}$ subtends to the arc DC and therefore $DC = \widehat{CGD} \cdot r = 2 \cdot \widehat{CAD} \cdot r,$ the last being so because the angle $\widehat{CGD}$ is central whereas the angle $\widehat{CAD}$ is inscribed to the circle, and they both subtend to the same arc.

However, we can understand that the arcs BC and DC are equal. So out of the previous equations we get:
$BC = DC$
$\widehat{CAB} \cdot 2r = \widehat{CAD} \cdot 2r$
$\widehat{CAB} = \widehat{CAD}$

As mentioned above, since the two angles are equal, the three points, A, D, and B, are part of the same line and we have successfully proven that any given point of the small circle will form a straight line throughout its rotation inside of the bigger circle.

CONCLUSION

In this paper, the centre of attention was the rotating circle. We examined two distinct problems, namely the Brachistochrone and the Tusi couple.

The solution to the Brachistochrone problem proved to be the cycloid. The cycloid is a fascinating curve with unique properties [2,4] and is created by the rotation of a circle on a straight line. In our presentation in Euromath 2018 in Krakow, Poland, we demonstrated the result to the Brachistochrone problem using a hand-made device of three wooden curves and three balls, inspired by [4]. Indeed, the cycloidal curve was the fastest one.

The solution to the Tusi couple proved to be the straight line. This was an unexpected result and, in the last section of our paper, we provided an interesting proof. There are many other similar brainteasers, for example, consider the situation where the smaller circle has one fourth of the diameter of the bigger circle. What curve would a point on the smaller circle create now as the small circle rotates inside the bigger one [4]?

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ELLIPSE: AN INCREDIBLE, YET OFTEN OVERLOOKED, CURVE

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ABSTRACT

The intersection of a plane with the surface of a cone creates four shapes, the conic sections. The Ellipse is one of them and although it is present in our everyday lives, we do not really pay attention to it.

Did you know that all planets of our solar system follow elliptical orbits? Or, that whispering galleries, the lithotripsy treatment and the elliptical pool all work thanks to the reflective property of an ellipse?

In our paper, we demonstrate methods to draw ellipses, specifically the trammel method, the use of the ellipsograph and an approach using two pins, a pencil and a piece of string. We discuss the characteristics of an ellipse such as the foci and the eccentricity. We prove the reflective property and mention instances in which ellipses appear in the real world.

INTRODUCTION

The ellipse is a conic section that is created when the surface of a cone is intersected with an inclined plane, as shown in Figure 1. The ellipse is defined as the set of all points whose sum of the distances from two fixed points is constant. The two fixed points are called “foci”. The ellipse is considered a generalization of a circle, which is a special type of an ellipse having both focal points joined so close together that they appear to be one.

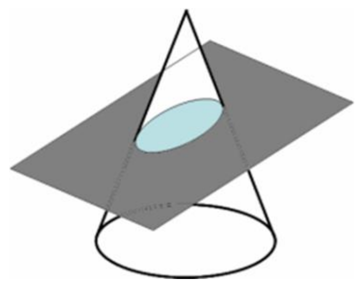

Figure 1: A cone intersected by a plane and creates an ellipse

The line through the two points of the ellipse, which are the furthest away from each other, is the major axis of the ellipse. The foci are points on the major axis. The middle point between the foci is the center of the ellipse. The line through the center, perpendicular to the major axis, which also connects the two points of the ellipse, which are the closest to each other, is the minor axis. The distance between the center and either focus is the fixed value c. The length of the semi major axis is the fixed value a. The length of the semi-minor axis is the fixed value b [1, 2].

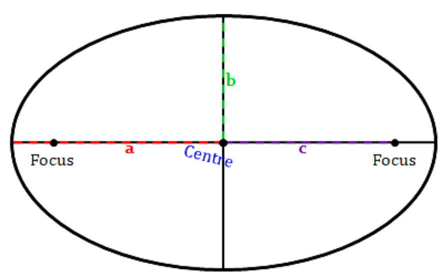

Figure 2: The Ellipse

The most known mathematicians to study the ellipse were four Greek mathematicians. Menaechmus (~380 BC-~320 BC) was the first one to investigate the ellipse (as well as the parabola and the hyperbola) as conic sections. After him, Euclid (~325 BC-~265 BC), who is best known for his work "Elements", also wrote about the ellipse. However, the name "ellipse" was given by Apollonius, "The Great Geometer" (~262 BC-~190 BC). Pappus (~290-~350) introduced the focus and directrix of an ellipse. The term "focus" was introduced by Johannes Kepler (1571-1630) much later, in 1609, when he published his discovery of Mars's orbit around the sun being elliptical with the sun at one focus, which led him to publish his first two laws of planetary motion in the same year and the third one in 1619 [3].

HOW TO CONSTRUCT AN ELLIPSE

There are many methods used to construct an ellipse. We present three: the Trammel Method, the Ellipsograph and a simple approach using two pins, a piece of string and a pencil (or any object that can trace the ellipse on the current surface).
In the Trammel Method, we firstly draw a pair of perpendicular lines, the longer one being the major axis and the shorter one being the minor axis, as shown in Figure 3. Then, we take a piece of paper and we mark the lengths of the semi-major axis (we name the point "a") and the semi-minor axis (we name the point "b"), as shown in Figure 4. By putting point a on the minor axis and point b on the major axis and marking where the edge of the paper is each time, as shown in Figure 5, we end up with points, all of which are points of that specific ellipse. If we join them with a curve, we obtain an ellipse.

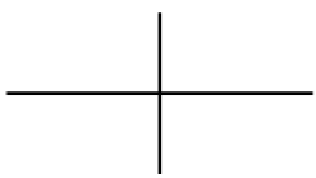

Figure 3

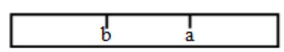

Figure 4

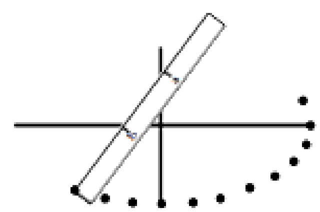

Figure 5

The Ellipsograph or Trammel of Archimedes is a device commonly used for drawing ellipses, which is basically a mechanized version of the Trammel Method. It consists of two small blocks (point a and b) which slide back and forth in their tracks (major and minor axis) with the help of a handle, as shown in Figure 6. At the end of the handle, there is a pencil, which traces the shape of the ellipse [2].

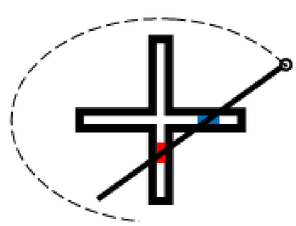

Figure 6: The Ellipsograph

Lastly, there is a very easy way to construct an ellipse, which requires only two pins, a piece of string and a pencil. We push the two pins in a surface and tie a piece of string around each one, long enough to be able to go around them. Using the pencil, we stretch the string the furthest it allows us and by dragging it, it traces an ellipse, as shown in Figure 7 [2].

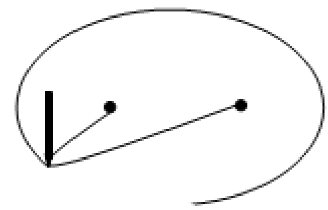

Figure 7: A simpler way to construct an ellipse

THE EQUATION OF THE ELLIPSE

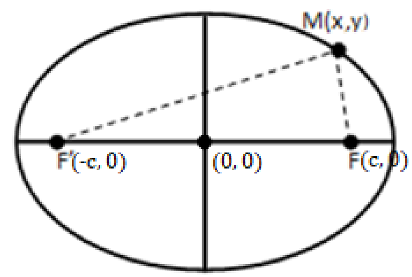

Figure 8: A random point M(x, y) of an ellipse and its distances from the foci.

In order to obtain the equation of the ellipse, we need the distances between a random point M(x, y) and each of the foci:

$MF = \sqrt{(x-c)^2 + y^2}$ (1) and

$MF' = \sqrt{(x-(-c))^2 + y^2} = \sqrt{(x+c)^2 + y^2}$ (2)

the sum of which, by definition, is equal to 2a:

$MF + MF' = 2a$ (3)

Equations (1), (2) and (3) yield

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

After squaring both sides, doing some basic mathematical operations and also replacing $a^2 - c^2$ with b^2 (since the relation that connects a, b and c is $a^2 = b^2 + c^2$), we obtain the equation of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ECCENTRICITY

The shape of an ellipse (how "elongated" it is) is represented by its eccentricity, which can be any real number from 0 (the limiting case of a circle) to arbitrarily close to, but less than, 1. The eccentricity is denoted by the letter "e" and is defined by the ratio:

$$e = \frac{c}{a}$$

where c is the distance from the center to the focus of the ellipse and a is the length of the semi-major axis. Due to the fact that $c = \sqrt{a^2 - b^2}$,

$e = \frac{\sqrt{a^2-b^2}}{a} \Leftrightarrow e^2 = \frac{a^2-b^2}{a^2} = 1 - \left(\frac{b}{a}\right)^2 \Leftrightarrow \frac{b}{a} = \sqrt{1 - e^2}$

We observe that the greater the eccentricity is, the smaller the ratio b/a becomes, which makes the ellipse more elongated. This happens because for the ratio b/a to become smaller, either b gets smaller or a gets greater (or both). If b gets smaller the ellipse loses width and if a gets greater the ellipse gains length, which both result in the elongation of the ellipse.

THE REFLECTIN PROPERTY

According to the Reflective Property, when a ray leaves one of the foci and meets a point of the ellipse, it reflects off the ellipse and passes through the other focus point. This happens because when a ray meets the surface of a curve, the angle between the incident ray and the tangent to that point is equal to the angle between the reflected ray and the tangent, as shown in Figure 9 [2, 5].

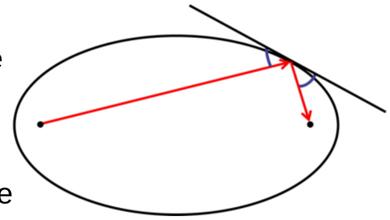

Figure 9: The reflective property

PROOF OF REFLECTIVE PROPERTY:

In order to prove the reflective property [4], we need to take a step back and prove that every point outside the ellipse is at a greater distance from its two foci than a point on the ellipse.
Let K be the point outside the ellipse and M the point on the ellipse as in Figure 10. Then:
(1) $\mathrm{KF1} + \mathrm{KM} > MF1$
(2) $\mathrm{KF2} = \mathrm{KM} + \mathrm{MF2}$
From (1) and (2) we obtain: $\mathrm{KF1} + \mathrm{KF2} > MF1 + MF2$. (3)

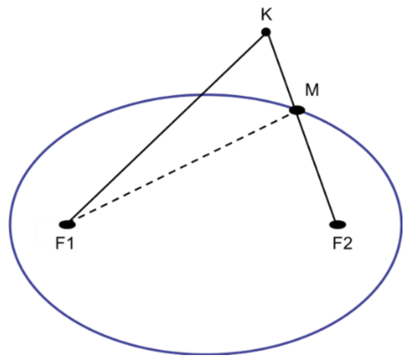

Figure 10: The reflective property; initial figure

Consider an ellipse with its two foci F1 and F2, as shown in Figure 11. We draw a random point of the ellipse, called M, and the tangent of the ellipse at the point M. By joining F1 with M we create the angle a. Then, we draw the symmetrical point of F1, called A, in relation to the tangent and, we join the points A and F1. Therefore, the line segment AF1 intersects the tangent at point E with $\mathrm{AE} \perp \mathrm{ME}$ and $\mathrm{AE} = \mathrm{EF1}$. We join the two points A and M and form two equal right triangles, AME and $\mathrm{EMF1}$ (because they have the common side EM and EA = EF1). Hence, the angle a is equal to the angle c.
Subsequently, we join the points M and F_2 and create the angle b. The only thing that remains to prove is that A, M and F2 belong to the same line.
Recall the inequality (3). In our case, comparing the distances of any other point, for instance N on the tangent, the sum will be greater since the said point is
outside the ellipse.
Knowing that $\mathrm{MF1} + \mathrm{MF2} = \mathrm{MA} + \mathrm{MF2}$, (which is constant), we conclude that $\mathrm{NF1} + \mathrm{NF2} > MA + MF2$, for all N on the tangent.

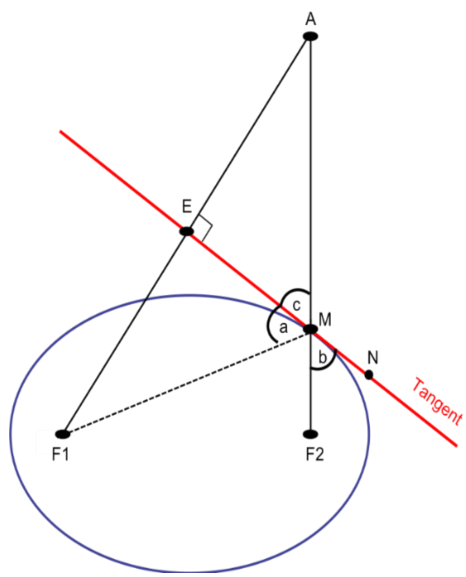

Figure 11: Proof of the reflective property.

For such an inequality to hold for all N we must have that A, M and F2 are collinear. Hence $\hat{c} = \hat{b}$, since there are vertiginous angles, and $\hat{a} = \hat{c} = \hat{b}$.

APPLICATION OF THE ELLIPSE: WHISPERING GALLERY

A whispering gallery is a construction based on the reflective property of the ellipse and on elliptical reflectors. The sound waves from someone who whispers at a focus point travel directly to the other focus point and what is said can be heard clearly by someone standing at that point but by nobody else. This happens because of the reflective property of the ellipse.
There exist several constructions that are examples of whispering galleries such as the National *Statuary Hall in Washington D.C.*, St. Paul's Cathedral in London and Gol Gumbaz Mausoleum in Bijapur, India. [5]

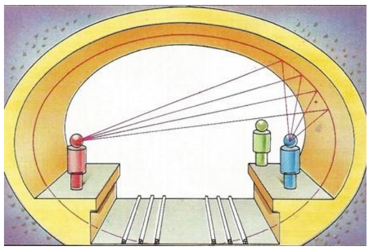

Figure 12: How Whispering Galleries work

Figure 13: St. Paul's Cathedral, London

APPLICATIONS OF THE ELLIPSE: LITHOTRIPSY

Lithotripsy is a medical procedure used to destroy kidney stones. The patient lies in a lithotripter machine of an ellipsoid shape, which can be considered as a three-dimensional generalization of an ellipse. The kidney stone, inside the patient's body, is positioned at one focus. In order to achieve positioning of the kidney stone at a focus, doctors use a fluoroscopic x-ray system to detect the position of the kidney stone. Shockwaves, which originate from the other focus, meet a point of the ellipsoid and are reflected and therefore are concentrated on the kidney stone. This way, the kidney stone breaks into little pieces which are able to pass through the patient's body without causing any pain [6].

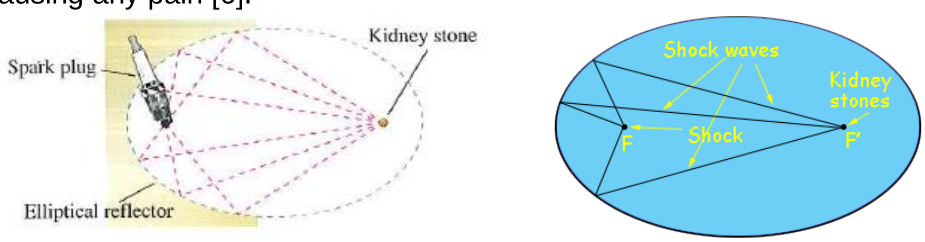

Figure 14: How lithotripsy works

APPLICATIONS OF THE ELLIPSE: ELLIPTICAL POOL

Elliptical pool is a game created by Alex Bellos, a British mathematician. Normally, pool tables are rectangular and have 6 pockets in the edges of the walls. In the elliptical pool table, called “Loop”, there is only one pocket, which is located exactly on one focus. There is also a dot on the other focus point. There are 4 balls, the black one placed on the dot, a yellow and a red one on either side and the standard cue ball, which is used to hit the other ones and which is positioned anywhere on the line between the black one and the pocket, as shown in Figure 15. The first player chooses one of the colored balls and he has to hit it. Once he has put the colored ball of his choice in the pocket, he can pot the black one. This way, he wins. The rest of the rules are the same as those of regular pool.

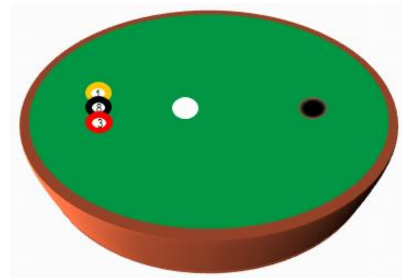

Figure 15: The starting position of each ball in elliptical pool

So, how do we play elliptical pool? As we know, thanks to the reflective property, if a ball is placed on the dot, which is a focus, it will definitely get in the pocket, the other focus, whichever direction we hit it (obviously by applying the suitable amount of force). But it is very rare for a ball to be exactly on the focus. Well, we hit the colored ball with the cue ball in a way that it would look like it was coming through the dot. So assuming that the red ball is on the spot shown in Figure 16, we imagine the line that it would connect its center to the dot. This way, we can accurately tell which spot of the red we must hit with the cue ball. It is the point that the line and the red ball touch each other. If we hit it on this exact spot, it will definitely be potted in the pocket [7].

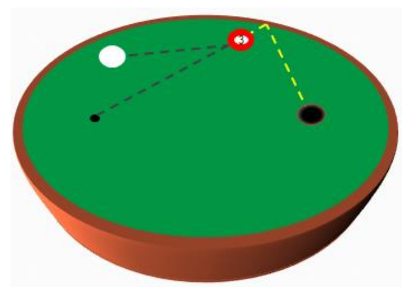

Figure 16: How to pot a ball that is not positioned on the dot.

KEPLER’S FIRST AND SECOND LAW OF PLANETARY MOTION

Kepler’s first law describes the movement of the planets around the sun. In short, the First Law of Planetary Motion states that all planets move in elliptical orbits around the sun, where the sun is positioned in one focus. Kepler realized this while he was observing planet Mars and noticed that the planet is not moving in circular orbits, which was thought to be the case, according to the model of Copernicus.

Earth’s orbit around the sun is almost a perfect circle, due to the fact that its eccentricity is only 0.0167. The most circular orbit is that of Venus, which has an eccentricity of 0.0068. On the other hand, Pluto’s orbit is the least circular one, with an eccentricity of 0.2488. (See Figures 17-19.)

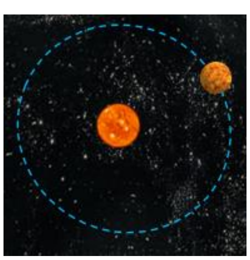

Figure 17: Venus’s orbit around the sun

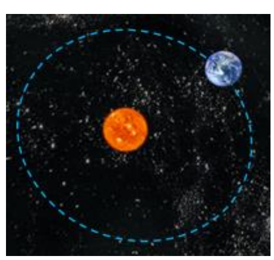

Figure 18: Earth’s orbit around the sun

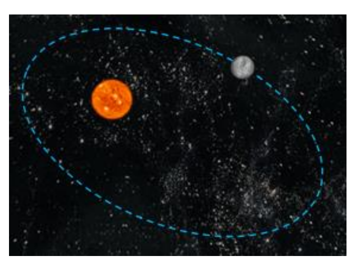

Figure 19: Pluto’s orbit around the sun

In his Second Law of Planetary Motion, Kepler explains how planets' speed is greater when they are closer to the sun than when they are further away, which results in different distances being covered in equal time periods. This is well explained in Figure 20, in which the three green sections cover the same area. The curved side of each section is a part of the planet's orbit, which Kepler realized that the planet travelled in equal times. Subsequently, while travelling a relatively longer distance, the velocity of the planet will increase and while travelling a relatively shorter distance, it will decrease [2, 8].

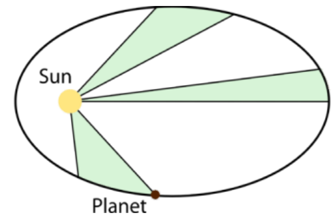

Figure 20: Kepler's Second Law of Planetary Motion

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P VERSUS NP

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ABSTRACT

In this work we study one of the most famous mathematical problems that is yet unsolved, namely P vs NP. The P vs NP question (one of the six millennium prize problems) regards the difference between the class of problems that can be solved in polynomial time by a deterministic algorithm and the class of problems that can be solved in polynomial time non-deterministically. Are these problems identical in terms of time complexity or not?
We present the history behind the problem, explain its importance in various domains (from science to art) and explore the various applications that its solution will provide. We then proceed by giving all the necessary definitions needed to describe the problem in a concrete way, we present some problems from both P and NP, while we also try to catch the essence of the difference of these two classes by also exploring their meaning from a philosophical point of view. Finally, we conclude by studying the reduction proof method, the main mathematical tool used in order to understand in which class each problem belongs.

INTRODUCTION

At the current work we examine the problem 'P vs NP', which is one of the most famous mathematical problems that is yet unsolved. This problem has great significance for both Mathematics and Computer Science and in addition is one of the Millennium Prize Problems. The Millennium Prize Problems are seven problems that were stated by the Clay Mathematics Institute in 2000. As their name states, if someone solves one of these problems, he would get one million dollars as a prize.
The other six problems are:

- Poincaré conjecture is solved by Grigori Perelman in 2003. His proof is based on work by Richard Hamilton; Perelman was selected to receive the Fields medal for his solution and he was officially awarded the Millennium Prize but he declined both awards.
- Hodge conjecture.
- Riemann Hypothesis.
- Yang – Mills existence and mass gap.
- Navier – Stokes existence and smoothness.
- Birch and Swinnerton – Dyer conjecture.

The outline of the current paper will be the following: we start by discussing briefly the history of 'P vs NP' question and explore why is so important. Then we give the necessary definitions of the notions needed to describe it while we also present some problems from both P and NP. These problems help to capture the essence of the difference between these two classes. Moreover, we give some philosophical aspects and applications that its solution will provide. Finally we present the reduction proof method which helps us to understand in which class each problem belongs and we finish by presenting the conclusions.

WHAT EXACTLY IS THE 'P VS NP' PROBLEM?

The P versus NP problem asks whether every problem whose solution can be quickly verified (technically, verified in polynomial time, see definitions below) can also be solved quickly (again, in polynomial time). This simple question is a major problem which remains unsolved until today. The informal term quickly, used above, means the existence of an algorithm solving the task that runs in polynomial time (P), such that the time to complete the task varies as a polynomial function on the size of the input to the algorithm (as opposed to polynomial time (NP), say, exponential time). For some questions, there is no known way to find an answer quickly, but if one is provided, with information showing what the answer is, it is possible to verify the answer quickly.
So the question is: Is P = NP or P ≠ NP?
An answer to the P vs NP question would determine whether problems that can be verified in polynomial time can also be solved in polynomial time or not. If it turned out that P ≠ NP, it would mean that there are problems in NP that are harder to compute than to verify: they could not be solved in polynomial time, but the answer could be verified in polynomial time.

A BRIEF HISTORY ABOUT THE 'P VS NP' PROBLEM

Although the P versus NP problem was formally defined in 1971 by Stephen Cook in his seminal paper "The complexity of theorem proving procedures", there were previous inklings of the problems involved, the difficulty of proof, and the potential consequences. In 1955, mathematician John Nash wrote a letter to the NSA (National Security Agency), where he speculated that cracking a sufficiently complex code would require time exponential in the length of the key. If proved (and Nash was suitably skeptical) this would imply what we today would call P ≠ NP, since a proposed key can easily be verified in polynomial time. Another mention of the underlying problem occurred in a 1956 letter written by Kurt Gödel to John von Neumann. Gödel asked whether theorem-proving (now known to be co-NP-complete) could be solved in polynomial time, and pointed out one of the most important consequences—that if so, then the discovery of mathematical proofs could be automated.

P PROBLEMS: DEFINITIONS AND EXAMPLES

Firstly, we examine what is the meaning of P problems and we give an example.
P problems:
P stands for Polynomial and its meaning is that it is a set that contains the problems that can be solved in polynomial time by a deterministic(?) procedure.
Most of the times we call this procedure an algorithm. In general, we use P in order to describe easy problems.

Polynomial time:
Suppose that you have a mathematical problem and you also have an algorithm that solves this problem, so you give the problem to the algorithm as an input and the algorithm takes this input and produces the solution.
Then, this input has a size which we express with a number, lets say n. Now the algorithm needs some time in order to solve the problem and this time can be expressed as a function of the size n of the input, so basically,
Time= f(n).
Let's give a very simple example:
Suppose that you want to multiply two numbers and also assume that the multiplication or the addition of two, 1 digit, numbers can be computed in constant time, which we call a step.
The question is 'How many steps are needed to multiply two numbers of more than 1 digits?'.
Let's try the algorithm that we know from elementary school!!!!!
We multiply 16, 12, two numbers of n=2 digits

```
   16
  x12
 ----
   32   (2 steps)
 +16    (2 steps)
 ----
  192   (1 step)
```

Total number of steps: 2+2+1=5=22+1.
In general, when we have to multiply two numbers of at most n digits each, the simple elementary school algorithm can produce the solution in at most cn2 steps, where c is some positive constant. Thus the time we need to multiply two n digit numbers can be described by a polynomial function, specifically f(n)= cn2.
So, as we can see this is quite interesting since when we were kids, although we were not aware of it, we learnt one of our first polynomial time algorithms!!!

NP PROBLEMS: DEFINITIONS AND EXAMPLES

NP problems:
NP stands for Non deterministic Polynomial and its meaning is that it is a set that contains the problems that can be solved in polynomial time by a non deterministic(?) algorithm. In general, we use NP in order to describe the hard-difficult problems.
To make it clear, let's give the example of security cracking:
Suppose that someone gives you the following problem:
There are n=3 black squares and behind each square is hidden a number from 0 to 9

The final 3-digit number represents the security code of a credit card, find the 3-digit number.
In this problem we notice that we do not know any clever way to solve the problem so, we have to take the brute force approach!

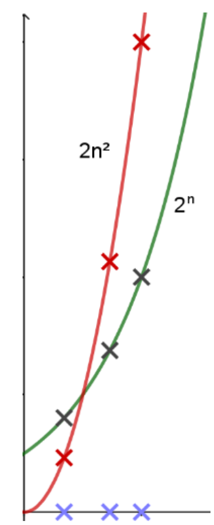
Let's start by exploring the investigation that fits right here:
Try all possible 3-digit numbers! Each try takes constant time and once again this constant time defines a step. How many steps we will do in the worst case?
Each square has one number from 0 to 9 (totally 10 possible numbers) and we have n=3 such squares and so, in total we will need 103=1000 steps.
In general, when we have to find a n-digit number in the decimal system, the most efficient way to find it is simply to try all possible n-digit numbers. This will take up to 10n steps and this number defines also the time that our algorithm needs to solve this problem. Now notice that f(n)=10n is not a polynomial function this time. In contrast, it is exponential!

THE FIRST CONCLUSION

Above, we saw a problem that can be solved in polynomial time and a problem that can be solved in exponential time. And the question is 'But which of them is faster?'.
Let's try to compare n3 with 3n for n=20. We notice that:
203=8000 steps when our input has size 20, while 320=10460353203 steps when our input has size 20!!!!!!!!
And the verdict is that:
The polynomial algorithms are much faster. When the only way to solve a problem is an exponential algorithm, as n grows, the problem becomes practically unsolvable!!!!!
If we take a look to the below diagram of the functions 2n2 and 2n, it's very clear that the polynomial function is faster of the exponential.

.

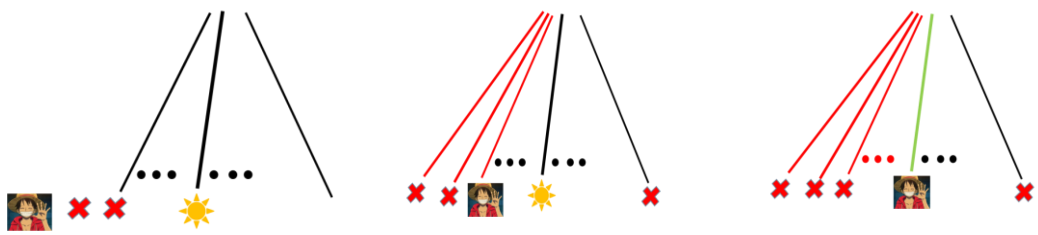
run of algorithm 1	run of algorithm 2
Algorithm 1 is $2n^2$ (a polynomial-time algorithm)	Algorithm 2 is not cn^k for any constant k (not a polynomial-time algorithm), but it is 2^n



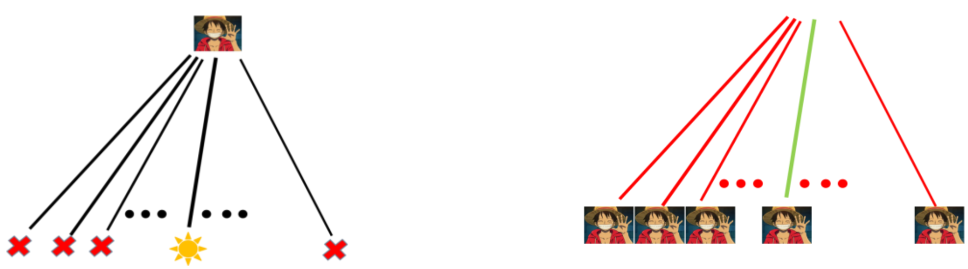
Polynomial time problems are easy to solve and because of that, calculators can compute multiplications of huge numbers in an instant. However, exponential time problems are almost unsolvable for large inputs and that is why it is not easy to crack bank security codes.

DETERMINISM – NON DETERMINISM DEFINITIONS AND THE DIFFERENCE

We saw how we define the 'time' a problem needs to be solved and we also saw what polynomial time means, as well as what is the difference with other classes of functions, like exponential. But what about determinism and what is the difference with the non determinism?
Let's start with determinism. The easiest way to think of this notion is to imagine how a human being approaches a problem or a situation. You usually make one step at a time. These steps follow a serial pattern. You are restricted by nature limitations.
For instance, Luffy wants to go to a certain destination, but does not know which route to take. As a result, he has to try each and every one of them in order to find the right one.



On the other hand, in a non deterministic universe you can forget about human beings and their limitations, you have the powers of a god. You can make multiple steps at a time and these steps can be done in parallel.
So let's return to the previous example and let Luffy once again search for the right route. This time he has the power to go to the destinations at once. This way, he can find the solution in an instant.



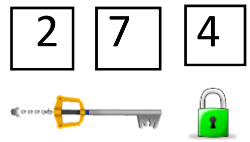
So, the difference between determinism and non determinism is that:
In a deterministic universe, given a problem you have to produce the solution and this might take some time. In contrast, in a non deterministic universe, given a problem you once again have to produce the solution, however this time you can produce it in an instant and you only need to verify if it is correct.

THE SECOND CONCLUSION

If we return to security cracking we notice that the best deterministic algorithm that we have, as we saw,

takes exponential time and this will take up to 10^n steps.

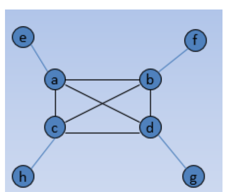
But what if we try to solve it non deterministically?
Since we have godlike powers in this case, we already know the solution and we only need to check if it is correct.

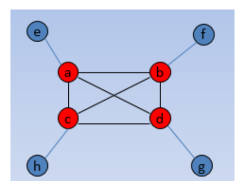


And what about 'time'? Just press the numbers and now we need n instead of 10^n steps!!!!!

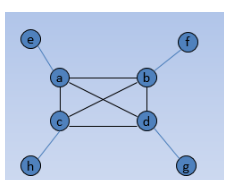
THE REDUCTION METHOD

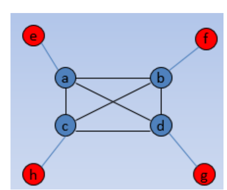
Here we present the reduction proof method which helps us to understand in which class each problem belongs. We will show that if we solve an NP-complete problem, then we can solve every NP-complete problem. This also implies that if we manage to solve a NP-complete problem in polynomial time deterministically then the same goes from every problem in this class.
We will describe this method using two examples, the example of clique and the example of independence set which both belong to the NP-complete class.
The example of clique: Given a graph, find the largest set of vertices that are connected to each other, namely,





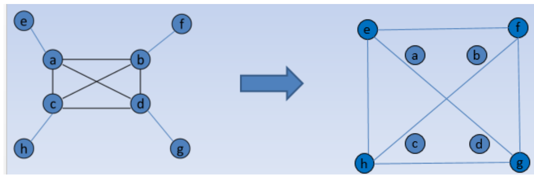
The answer is that the largest set of vertices has four vertices (a,b,c,d with red color).
The example of independence set: Given a graph, find the largest set of vertices that are not connected to each other, namely,



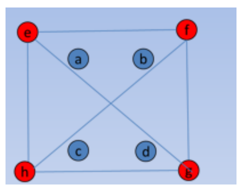


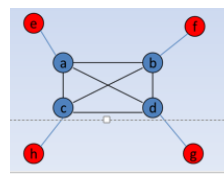
The answer is that the largest set of vertices has four vertices (e,f,g,h with red color).
How we will prove that the two problems are equivalent?

We start from one problem and go to the other. Suppose we have an algorithm that solves the clique. Then the algorithm solves the problem in each graph, thus it also solves it both in ours and in its complementary, where complementary is called the graph that does not have the edges of the original but has those that did not exist in the original.



We solve the clique problem in the complementary graph then this solution is a set of independence in the original graph !!!





What exactly happened?
Each set of vertices which are connected to each other in the complementary graph, due to its definition, are not connected to the original graph. The maximum complementary of clique is always the maximum set of independence in the original!!! By doing this conversion, each time we can solve one problem then we can solve the other. And so, the two problems are equivalent !!!

A QUESTION AND AN ANSWER

To sum up, we have that:
P: The set of problems that we can produce the solution in polynomial time.
NP: The set of problems that if we could produce the solution in an instant (or if we basically knew the solution), we could check if it is correct in polynomial time.
But there is one more question which is very important:
What is more difficult?
To produce the solution of a problem in polynomial time or to check in polynomial time if the solution is correct (given that we already know it)?
Let's rephrase the question for being a bit informal.
Question: Do we really have limitations and some problems can not be solved fast enough or we can solve even the problems that we believe that are unsolvable in terms of time?
Are we really restricted by our human nature or we can actually achieve a state far beyond that?
A state where no limitations exist, a state where no matter what we face, there is no real challenge?
Answer: We do not know…Although we compare humans with something much more powerful, although most of the mathematicians believe that P and NP are not equal, we can not prove it.

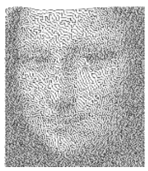
THE EFFECTS OF THE 'P VS NP' ON ART

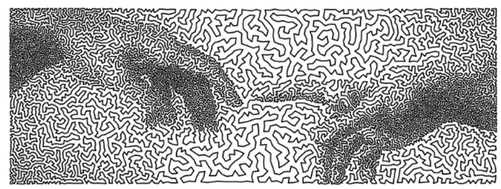
Aside from being an important problem in mathematics and computer science, a proof either way would have profound implications for mathematics, cryptography, algorithm research, artificial intelligence, game theory, multimedia processing, philosophy, ecomics and many other fields.

'P vs NP' problem has influenced many fields of art.

One famous problem which belongs in the class of NP problems is the travelling salesman problem (TSP). TSP asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?" . This problem has influenced the art of design, cinema and literature.

In "TSP art" an image's tonal quality is reproduced with a single, continuous path. This single path meanders over the entire canvas.

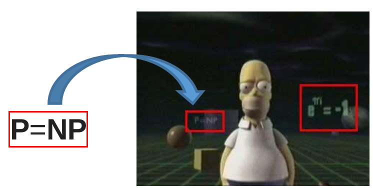




The thriller novel The Steradian Trail by M. N. Krish weaves the traveling salesman problem and mathematician Srinivasa Ramanujan and his accidental discovery into its plot connecting religion, mathematics, finance and economics.

The film Travelling Salesman, by director Timothy Lanzone, is the story of four mathematicians hired by the US government to solve the P versus NP problem.

In the sixth episode of The Simpson's seventh season 'Treehouse of Horror VI', the equation P=NP is seen shortly after Homer accidentally stumbles into the "third dimension".



In the second episode of season 2 of Elementary, 'Solve for X' revolves around Sherlock and Watson investigating the murders of mathematicians who were attempting to solve P versus NP.

CONCLUSION AND A PHILOSOPHICAL ASPECT

As we saw, there is no answer in the question 'is P and NP problems identical?'.

However, most scientist believe that these sets are not identical. But there is another question which is important:

'what P=NP would mean if that was the case?'

The American Theoretical computer scientist Scott Aaronson has wrote:

'The world would be a profoundly different place than we usually assume it to be. There would be no special value in 'creative leaps', no fundamental gap between solving a problem and

recognizing the solution once it's found. Everyone who could appreciate a symphony would be Motzart; Everyone who could follow step by step an argument would be Gauss.'
Frédéric Chopin once said the following which could be an answer to the above question : 'Simplicity is the final achievement. After one has played a vast quantity of notes and more notes, it is simplicity that emerges as the crowning reward of art'.
Similarly, Jack Kerouac has said:
'One day I will find the right words, and they will be simple'.

AKNOWLEDGMENTS

We would like to thank our supervisor Dr. Sophia Birmpa – Pappa for her help and support during the preparation of this work and, also, our classmates and members of Zanneio Club of Mathematics 'Maths in Reality' for their ideas.

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MATHEMATICAL FALLACIES

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The English School Nicosia, Cyprus

ABSTRACT

As Galileo Galilei said, the universe cannot be read until we have learned its language and become familiar with the characters in which it is written. This is the Mathematical Language! This 'language' is indeed Mathematical Language and And the 'letters' are the infinite numbers, curves and other geometrical figures. It is Mathematics that directs the flow of the universe, a precise model of real phenomena and an explanation to common fallacies, we, as human beings have. Imagine that you are watching the end of a long action movie. The hero and villain are in a mad chase over a city skyline to deactivate and trigger a nuclear bomb respectively. The villain throws himself down on a curved building and the hero throws himself into the air and hits a lower part of the curve with the intent to slide to the bottom faster. Both however, take the same time to reach the end of the curve. How, and why is that? Well, this is due to the Tautochrone curve theorem, proved by Huygens back in the 17th century. Yet as far as the laws of this ingenious language refer to reality, in fact its numerical letters are not certain, and as far as they are certain, they do not refer to reality. We tend to look at all numbers as fixed finite figures, without questioning why. Why is it that we take $0.\dot{9}$ to be 1, or why 1 may be equal to 2? Well, all this can be proved!

INTRODUCTION

In this report, we investigate various everyday situations that we have mistakenly believed to have fully understood. We will in fact examine a number of ideas that a lot of people think to be true, but which are in fact false. In trying to do this, we need to consider that to understand the universe and question things in it we must, as Galileo Galilei said, understand its language, the mathematical language.

You see, maths is all about uncertainty and reality, proving why ideas might be 'invalid' and the hidden truth behind what we think is correct. FALLACIES. For example, we delve into a situation where various similar-surfaced objects are projected from different parts of a cycloid curve and calculate as well as compare the time taken for each one to reach the minimum point of the curve. Furthermore, we look into how arguments about numerical theories come about and counter argued, backed with mathematical proofs. Specifically, we examine whether 1 can be considered equal to 2, or whether it is valid to say that 1 is equal to 0.9 recurring.

TAUTOCHRONE THEOREM

Imagine you are watching the end of a long action movie. The hero and the villain are in a mad chase over a city skyline trying to deactivate or trigger a nuclear bomb respectively. The villain throws himself down on a cyclical curve on the side of a building and the hero throws himself into the air and lands on a lower part of the curve with the intent to slide to the bottom faster. Who will reach the bomb first? Eventually they will both reach the bomb at the same time. Why is that? This is due to the Tautochrone theorem proved by Christiaan Huygens back in the 17th century.
In our attempt to prove this perplexing result and reach this conclusion we use the theory of a simple pendulum.

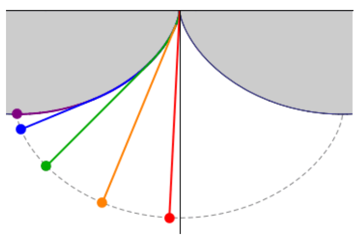

Figure. 1.1

Figure. 1.2

A pendulum consists of a bob of certain mass suspended from a chord moving back and forth following a cyclic path.

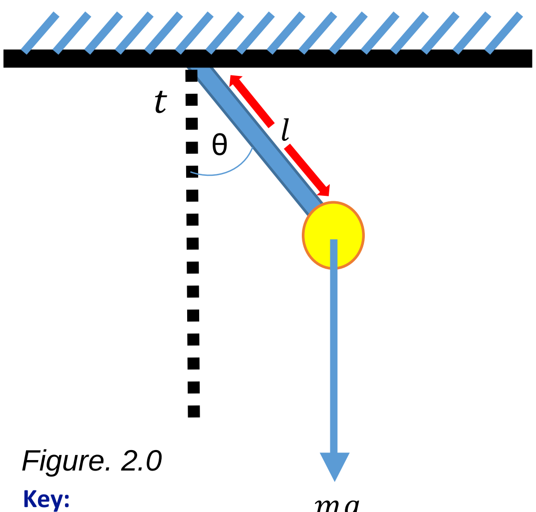

Figure. 2.0

Key:

- $Let\ t = tension\ [N]$
- $Let\ \ m = mass\ [Kg]$
- $Let\ W\ = weight\ = mg[N]$
- $Let\ F = force = [N]$

A pendulum bob of a certain mass, suspended from the end of a light weight cord at an amplitude θ where θ is the angle that the cord makes with the vertical equilibrium position. The bob is at a certain length from the pivot, which is the length of the cord, represented with the letter "l".

However, some assumptions must be made for valid results.

Chord:
(i) the length of the chord must be inelastic meaning that it should not change throughout its swing.
(ii) the mass of the chord must be constant and thus can be ignored relative to that of the bob.
Friction:
Friction caused by the motion of the pendulum moving back and forth is negligible.

Angle θ:
The amplitude of a pendulum bob from the position of equilibrium which is angle θ is small enough so that $\sin\theta$ is approximately equal to θ, however, only when using radians

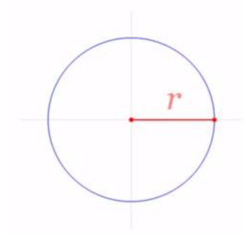

Figure. 2.1

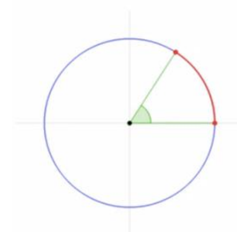
Figure. 2.2

The amplitude - angle θ - is small enough so that sinθ ≈ θ, only when using radians.

Radians:

- *unit of measurement of angles* ≈ 57.3°
- *1 radian = 1 arc of a circle with the length of its radius.*

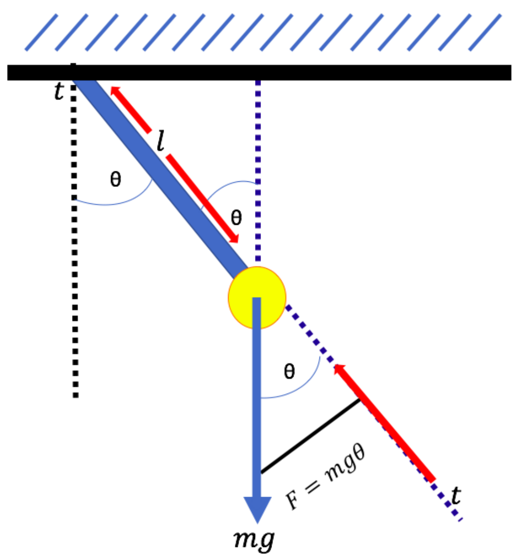

Figure. 3.0

According to Newton's 2nd law of motion, if the forces acting on an object are unbalanced then the object will accelerate or decelerate.
$Force = mass \times acceleration$
when the bob is suspended away from the equilibrium position, it accelerates as its mass is being attracted to the centre of the earth.

In this way, the downward force (the weight) is the only force acting on the bob when it is hanging freely, assuming however that the friction created by the bob moving is negligible. Therefore, acceleration is equal to the gravitational field strength on every planet we are on. Thus, we get $a = g$. As a result, we deduce that the downward force is equal mg.
However, what causes the pendulum bob to swing is the force which is perpendicular to the tension. (Note that the tension is parallel to the cord).

To calculate the force, we use trigonometry as a right-angle triangle is formed between the mg, the tension and the perpendicular force to be calculated. So, we can use the angle between mg and tension which is corresponding to the angle θ and create the equation

$$sin\theta = \frac{perpendicular\ force}{mg}$$

then we rearrange the equation and we get that

$$Force = mgsin\theta$$

which by using our initial assumption that $sin\theta \approx \theta$, can be expressed as

$$Force = mg\theta$$

However, as discovered previously, $Force = mg\theta$, therefore substituting this equation into the Newton's equation above we get that

$$a = \frac{mg\theta}{m}$$

The masses are cancelled out $\therefore a = g\theta$

We know that $g\theta = -w^2y$, where y is the displacement from the equilibrium position and w is the angular velocity. Hence, now we can express θ in terms of $l\ and\ y. y$ forms a horizontal perpendicular line from the pendulum bob to the vertical equilibrium line. Thus, again using trigonometry $n\theta = \frac{y}{l}$, and according to the initial assumption that $sin\theta \approx \theta$, $\theta = \frac{y}{l}$.

Back to the equation that $g\theta = -w^2y$, we substitute $\theta = \frac{y}{l}$ and end up with $g\left(\frac{y}{l}\right) = -w^2y$. We cancel out the minus sign as it only symbolises direction, thus leading to the equation $g\left(\frac{y}{l}\right) = w^2y$

We know that $\omega = \frac{2\pi}{T}$ where T is the time of the pendulum swing, called a period and ω is the angular velocity. Squaring both sides, we get
$\omega^2 = \frac{(2\pi)^2}{(T)^2}$ and since $\omega^2 = \frac{g}{l}$ $\therefore \frac{g}{l} = \frac{(2\pi)^2}{(T)^2} = \frac{4\pi^2}{T^2}$.
Rearranging in terms of T,

$$T^2 = \frac{4\pi^2 l}{g}$$

And taking the root of both sides, leaves us with the formula of a period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

which shows that the Period T is completely independent of the amplitude θ as it “cancels out” in the previous calculations and it is also independent from the mass, however, only if the assumptions are maintained. Hence, we can see that the amplitude of an object hanging from a cord can vary without affecting the time taken for one complete swing from any starting point. And just as angle θ varies without affecting the Period, in the same way, in the Tautochrone theorem, masses released from various points on a curve (remember the hero and the villain) do not affect the time taken for the mass to reach the point of equilibrium on the curve, this is also present in other everyday situations, like children swinging on a swing or like an antique clock keeping a constant time.

THE FALLACY OF NUMBERS
1=2

Initially, at a first glimpse, the obvious answer to the absurd question whether 1 can be considered equal to 2, is negative. But what if this could be proven wrong?
If we assume that x=1, we can derive the following number 1 proof:
We could also approach this from another point of view, if we assume that α=β

$$\text{Let, } x = 1$$
$$x = x^2$$
$$x - 1 = x^2 - 1$$
$$\frac{x-1}{x-1} = 1 \longleftarrow \frac{x-1}{x-1} = \frac{x^2-1}{x-1} \longrightarrow \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$
$$1 = x + 1$$
$$\text{since, } x = 1$$
$$1 = 1 + 1$$
$$\therefore 1 = 2$$

Figure.4.1 taking $x = 1$, 1=2

$$Let\ \alpha = \beta$$
$$\alpha^2 = \alpha\beta$$
$$\alpha^2 - \beta^2 = \alpha\beta - \beta^2$$
$$\cancel{(\alpha - \beta)}(\alpha + \beta) = \beta\cancel{(\alpha - \beta)}$$
$$\therefore \alpha + \beta = \beta$$
$$Since, \alpha = \beta$$
$$\beta + \beta = \beta$$
$$2\beta = \beta$$
$$\text{Let, } \beta = 1$$
$$\therefore 1 = 2$$

Figure.4.2 Taking $\alpha = \beta$, and $\beta = 1$, $1 = 2$

In both cases, we get this impeccable result that 1 CAN actually be equal to 2!
But one should question oneself. We urge you to examine what we have just shown you. Do you really believe after this that 1 is actually equal to 2, even though there is a reasonable way to prove that? Isn’t this incongruous idea beyond the bounds of possibility? Why have you believed us?
We have just committed a crime in the sacred name of Mathematics!

In both cases

Let, $x = 1$	$Let, \alpha = \beta$
$x = x^2$	$\alpha^2 = \alpha\beta$
$x - 1 = x^2 - 1$	$\alpha^2 - \beta^2 = \alpha\beta - \beta^2$
$\frac{0}{0}$: $\frac{x-1}{x-1} = \frac{x^2-1}{x-1}$	$(\alpha - \beta)(\alpha + \beta) = \beta(\alpha - \beta)$ $\frac{2}{0}$
$1 = x + 1$	$\therefore \alpha + \beta = \beta$
since, $x = 1$	$Since, \alpha = \beta$
$1 = 1 + 1$	$\beta + \beta = \beta$
	$2\beta = \beta$
	Let, $\beta = 1$
$\therefore 1 = 2$	$\therefore 1 = 2$

Figure. 4.3

Referring to *Figure. 4.3*, as you can see, we have obviously divided by 0 in both cases, which clearly makes this an invalid proof. Haven't you noticed? Even Siri in *Figure. 5.0* cannot find a real solution to this division.

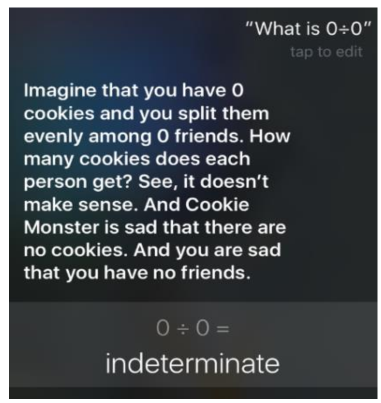

Figure. 5.0

As SIRI said ,

$\frac{0}{0}$ is indeterminate

$$1(0) = 2(0)$$

$$1 = 2 \quad \textbf{Fallacy}$$

Think of multiplication…

$$4 \times 10 = 40$$

$$4+4+4+4+4+4+4+4+4+4 = 40$$

Think of division…

$$25 \div 5 = 5$$

$$25 - 5 - 5 - 5 - 5 - 5 = 0$$

Figure. 5.1

Let's take it from the beginning. 1x0=0 which is also equal to 2x0. But if we assume that division by 0 is actually possible, we get this ridiculous fallacy that 1=2. Referring to *Figure. 5.1*, think of multiplication as glorified addition. And think of division which is glorified subtraction.

Consider, $\frac{5}{0}$

5-0-0-0-0-0-0.......

$$\frac{5}{0} = \infty \ ?$$

Figure. 5.2

But what happens when we divide 0? Let's consider $\frac{5}{0}$. As you can see in *Figure. 5.2,* it is 5-0-0-0-0-0-0 and it goes on. There is no end to this subtraction. So, wouldn't it make sense to say that $\frac{5}{0}$ is equal to infinity?

As a number is being divided by a smaller denominator the answer gets greater and greater. For instance, $\frac{1}{1}$=1 *,* $\frac{1}{0.1}$=10 and it goes on. Therefore, as the denominator decreases by the factor of x0.10 and gets closer to 0 the answer increases by the factor of x10. However, since a denominator can be decreased an infinite number of times in this way and never reach 0, the answer increases infinitely. Hence, division by 0 is equal to infinity. Taking another number, $\frac{2}{1}$ =2, $\frac{2}{0.1}$ =20 and it goes on. Thus, it would also be correct to say that division by 0 is equal to infinity. Since $\frac{1}{0}$ is equal to infinity, $\frac{2}{0}$ is equal to infinity as well. However, assuming that division by 0 is actually possible, we get this fallacy that 1=2. Therefore, we say that $\frac{1}{0}$ and $\frac{2}{0}$ are undefined.

$$\frac{1}{1} = 1 \qquad \frac{2}{1} = 2$$
$$\frac{1}{0.1} = 10 \qquad \frac{2}{0.1} = 20$$
$$\frac{1}{0.01} = 100 \qquad \frac{2}{0.01} = 200$$
$$\frac{1}{0.001} = 1000 \qquad \frac{2}{0.001} = 2000$$
$$\frac{1}{0.0001} = 10000 \qquad \frac{2}{0.0001} = 20000$$
$$\downarrow \qquad \downarrow$$
$$\frac{1}{0} = \infty \ ? \qquad \frac{2}{0} = \infty \ ?$$

Means that:

$$\frac{1}{\cancel{0}} = \infty = \frac{2}{\cancel{0}}$$

$$1 = 2?$$

$$\frac{1}{0} \ and \ \frac{2}{0} \ are \ undefined$$

Figure. 6.0

Previously, we attempted to go downwards towards 0 leading us to positive infinity. But there is another way to approach 0, going upwards towards 0, by dividing $\frac{1}{-1}$ = -1, $\frac{1}{-0.1}$ =-10 and it goes on, thus leading us to negative infinity.

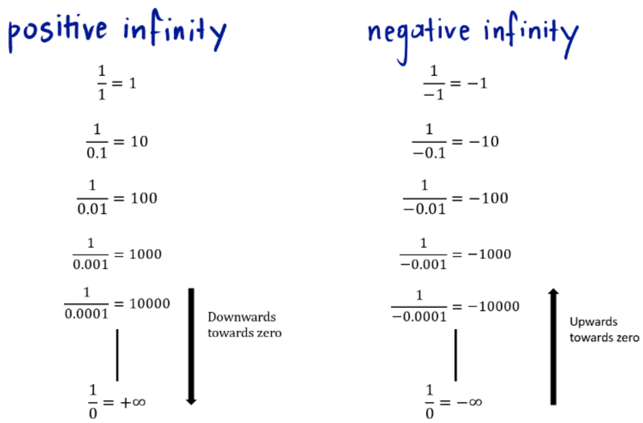

Figure. 6.1

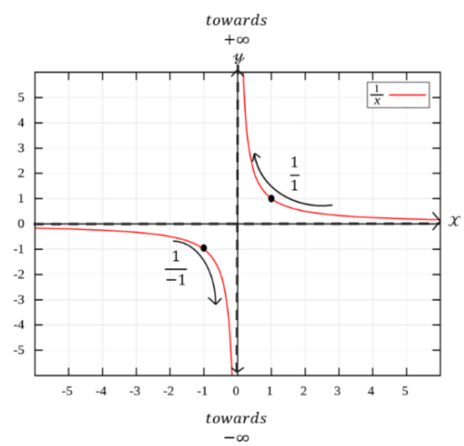

Figure. 7.0 Graphical representation as interpreted by the reciprocal function $\frac{1}{x}$. On the right side, we're going downwards towards 0 approaching positive infinity and on the left side we're going upwards towards 0 approaching negative infinity. As you can see x cannot be divided by 0. Therefore, both axes are asymptotes to the function, showing us that division by 0 is not possible.

To conclude, division by 0 is undefined. It cannot be defined or worked out, it is UNDEFINABLE. There is no number that you can pick which makes sense regarding infinity. Remember, infinity is not a number, but an idea. Therefore, division by 0 is indeterminate. Therefore 1 is NOT equal to 2.

$0.\dot{9}=1$

Despite all these lies, this time, that this fallacy will be valid. No tricks, no illusions, nothing surreal. This is a fallacy very simply explained.

$$\textit{let } x \textit{ be equal to } 0.\dot{9}$$

$$10 \times x = 10x \longleftarrow x = 0.\dot{9} \longrightarrow 10 \times 0.\dot{9} = 9.\dot{9}$$

$$\textit{multiply both sides by } 10$$

$$10x = 9.\dot{9}$$

$$\textit{subtract } x = 0.\dot{9} \textit{ from } 10x = 9.\dot{9}$$

$$\begin{array}{r} 10x = 9.\dot{9} \\ -\quad x = 0.\dot{9} \\ \hline 9x = 9 \\ x = 1 \end{array}$$

$$\textit{Since, } x = 0.\dot{9}$$

$$\therefore \mathbf{0.\dot{9} = 1}$$

Figure. 7.1

With reference to Figure. 7.1, this time we can really prove that $0.\dot{9}$=1.

This time, no tricks are involved. Nothing is hidden! Multiplying by 10 is allowed. Subtracting $0.\dot{9}$ from both sides is allowed. Dividing by 9 is allowed. Everything is allowed because everything is consistent. Therefore, $0.\dot{9}$ IS 1.

2

$$\frac{1}{9} \textit{ is equal to } 0.\dot{1}$$

$$\frac{1}{9} = 0.\dot{1}$$

$$\textit{multiply both sides by } 9$$

$$\frac{1}{9} \times 9 = 0.\dot{1} \times 9$$

$$\therefore 1 = 0.\dot{9}$$

3

$$\frac{1}{3} \textit{ is equal to } 0.\dot{3}$$

$$\frac{1}{3} = 0.\dot{3}$$

$$\textit{multiply both sides by } 3$$

$$\frac{1}{3} \times 3 = 0.\dot{3} \times 3$$

$$\therefore 1 = 0.\dot{9}$$

Figure. 8.0 alternative methods proving that $0.\dot{9}$=1

Taking any two consecutive numbers for example 1 and 2 you will notice the existence of an infinite number of decimals in between them, for example 1.2, 1.3, 1.897 etc. However, $0.\dot{9}$ and 1is one of the special occasions where no other number is found in between these two. In the subtraction of $0.\dot{9}$ from 1 the answer would be 0.00000… and it goes on forever. No number 1 can be found in this never-ending chain of zeros! The answer is so infinitely small, that for us it is simply 0. It is an infinitesimal, a thing so small that there is no way to measure it! And this does not mean that it is close to 0. But in fact, that IT IS 0!!! It is not that mathematicians have not figured this out, but just for the sake of simplification 0.9 recurring can be considered equal to 1.

CONCLUSION

In our report, we have discussed some of the most thought provoking fallacies that have long puzzled the human mind. We have talked about:

1. The Tautochrone theorem and its answer to the fallacy we used to have that one of the hero or the villain will reach the bomb at the minimum point of the curved building first, when starting from different positions.
2. The fallacy of numbers, including whether 1=2, where we discussed illicit proofs that led to the absurd theorems but in which the errors were not immediately apparent, and whether $0.\dot{9}$=1

Questioning things around us is a necessary quality that everyone should have. Be curious. Be inquisitive. Be inquiring. Because, as you must have realised, it is those who dig deeper and allow their minds to be puzzled, intrigued and stimulated that really understand fully how things operate and why. It is necessary to question things. It is okay to search for a logical explanation to everything. Things may seem strange at the beginning, but only up to the point that you begin to understand. And thereafter it is all mathematics.
MATHS IS THE HEART OF HIDDEN REALITY.

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WONDERWOMEN

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ABSTRACT

Mathematics as a science have existed for a period of approximately 2500 years and throughout this period many people have had a significant effect in their development. Most of us are indeed aware of prominent mathematicians such as Archimedes, Newton, Gauss and others. However, what most of us seem to be unaware of is the fact that very few women mathematicians enjoy similar levels of recognition. This does not mean that Mathematics were shaped into the science we know today by men only, as a large number of women have also added their inputs and ideas and have assisted in the creation of modern day Mathematics. In this paper we have selected four women mathematicians who have contributed greatly to their discipline and we will present their celebrated work as well as their extraordinary lives.

Throughout history, many extraordinary women passionate about STEM fields have left their mark. From mathematician Hypatia of Alexandria in the 4th century B.C. to many other women in the 21st century, women connected only by their insatiable interest in sciences have fought against vast cultural obstacles to become an inspiration for us all. We have decided to focus our research on four of these women who we believe have made a considerable impact in the world of Mathematics. Emmy Noether, Katherine Johnson, Shafi Goldwasser and Maryam Mirzakhani are all cases of exceptional women, who, against stereotypes, managed to prove that the power of the mind is not a matter of sex, and have become an ideal model of genius and willpower, especially for all of us, who want to pursue a career in science. Despite their achievements though, we are not sure their names are familiar to most people. It is our hope that through this presentation their contribution to Mathematics and Science in general will receive more attention and their accomplishments will inspire more and more young women to follow in their footsteps.

EMMY NOETHER

Amalie Emmy Noether was born in Erlangen, Germany, on the 23rd of March 1882. Her family was Jewish and her father was a mathematician at the University of Erlangen. She attended the same university as a student where she studied mathematics. Her final grades were high enough that she was automatically awarded the equivalent of a bachelor’s degree and started working at the Mathematical Institute of Erlangen. She worked there for almost a decade but was not paid for her work since women at the time were not allowed to hold academic positions. She was then invited to the University of Göttingen but objections from members of the existing faculty forced her to publish papers and give lectures under David Hilbert's name instead. It was during this period that Einstein published his general theory of relativity (1915). Noether applied her work to Einstein's theory and as a result she ended up with what we now acknowledge as Noether’s theorem. She stayed at the University of Göttingen until 1933 as a prominent mathematician. A large part of her work and ideas were described in Dutch mathematician B. L. van der Waerden's “Moderne Algebra” which was published in 1924. In 1933, as the Nazi party gained power in Germany, Noether along with other university professors were forced to abandon their positions. She then moved to Pennsylvania in the U.S.A. and took up a position at Bryn Mawr College. She passed away in 1935, at the age of 53, after she underwent surgery for an ovarian cyst.

Noether's main areas of focus were non commutative algebra, linear transformations, and commutative number fields. She is known for laying the foundations of abstract algebra as well as for her contribution to theoretical physics. She formulated her groundbreaking theorem, named after her "Noether's theorem", in which she states that every differentiable symmetry of the action of a physical system has a corresponding conservation law. In layman's words she said that wherever you find some sort of symmetry in a system you will also find a conservation of some quantity, for instance of momentum, or of energy or something else. Noether’s theorem in fact demostrates that a symmetry of time (time translation symmetry) can be linked to the conservation of energy. In simpler words, Noether’s theorem explains how energy can be neither created nor destroyed but merely changes form.

Noether also worked on ring theory (1920-1926), which is an area of abstract algebra. Rings are sets of elements in which addition and multiplication are possible between the elements contained in these sets, meaning that the resulting element would still belong in the set. Ring theory, as the name suggests, is the study of rings, meaning the study of the structure of rings and the properties of rings. However, Noether's contribution to Algebra and specifically to the commutative ring theory is quite abstract. This makes it hard to pinpoint her exact contribution, especially to anyone who does not yet possess a decent knowledge of algebra. But it was through Noether's work, that the connection between rings came to light. This in turn allowed us to provide answers for problems on all "alike" rings and algebraic structures, effectively solving a problem only for one of the rings and transferring the result to all alike rings.

Some rings were even named after her, as she was the one who studied them. Noether noted an attribute in certain types of rings (named "Noetherian Rings" in her honor) that are commonly studied in many mathematical fields and came up with a definition useful for distinguishing this specific type of rings. Noetherian rings are important in both commutative and non commutative ring theory. Nevertheless, trying to document and clarify Noether's contribution to mathematics remains at large an elusive hope. It is perhaps fitting to say that trying to study abstract algebra without Noether's input would be similar to trying to study and compare the grammatical structure of phrases without knowledge of the notions of subject, object, verb and other fundamental terms or to try to study triangles without knowing trigonometry.

KATHERINE JOHNSON

Katherine Johnson is one of the most notable mathematical figures of the 20th century. She has contributed greatly to the United States aeronautics and space programs and was instrumental in many of NASA's early projects such as Project Mercury, the Space Shuttle Program and various Apollo flights. Her main responsibility was calculating the trajectories of space flights but she also helped in many other areas of the programs as well. Katherine Johnson's contributions however, were brought to attention only recently through the release of the movie "Hidden Figures" in 2016. What makes her achievements even greater is the fact that in order to win the respect of her colleagues and the scientific community she had to overcome prejudices concerning both her gender and - more notably - the fact that she is African-American. Katherine Johnson retained her pride and dignity throughout her professional career and despite the odds stacked against her. In the end, it was the high quality of her work that made her invaluable to her team and also made it impossible for people to ignore her because of her skin color. Today she has received her due accolades and the respect she deserves for her efforts in the advancement of science.

Katherine Johnson had decided early in her life on becoming a research mathematician even though this was a difficult field for African Americans and especially for women. Consequently, after her graduation summa cum laude from college with degrees in Mathematics and French in 1937, Katherine initially worked as a teacher. During this time her aim was to “advance the race” by not just using book knowledge but also by inspiring discipline and self-respect in her students. She worked as a teacher until 1952 when she found out, from a relative, that the National Aeronautics and Space Administration (NASA) was hiring mathematicians. One year later she was offered a job and she became part of the early NASA team. Katherine spent the rest of her professional career at NASA and eventually retired from NASA in 1986.

From 1953 to 1958, Johnson worked as a "computer", analyzing topics such as gust alleviation for aircrafts. In mid-1960s Katherine, along with a colleague of hers, focused on the topic of computer failure. They formulated possible problems and backup solutions and finally developed simple navigational methods for the astronauts to use in case they lost contact with ground control. From 1958 until her retirement in 1986, Katherine worked as an aerospace technologist, at the Spacecraft Controls Branch. She calculated the trajectory for the May 5, 1961 space flight of Alan Shepard, the first American in space. When NASA used electronic computers for the first time in order to calculate John Glenn's orbit around Earth, Glenn refused to fly unless Katherine verified the calculations. At a later age, Katherine worked directly with digital computers. Her ability and reputation for accuracy helped to establish confidence in the new technology.

One of the important problems NASA scientists were faced with when trying to send the first rocket into space was trying to pinpoint where on the surface of our planet would a spaceship return after completing its travel. This was necessary both for collecting data from the actual flight as well as recovering the person or persons manning the spaceship. Katherine Johnson calculated the trajectory for Alan Shepard, the first American in space. An important aspect of this problem was determining the angle between the spaceship's trajectory and the earth's atmosphere at the point of reentry. Too steep and the spaceship would be destroyed by its high velocity, too shallow and the spaceship would "bounce off" the atmosphere and wander into space. So the basic question was calculating the exact position over the Earth to fire the retrorockets in order to land in the center of the ocean recovery zone.

Other aspects of the problem that would have to be taken into account were the earth's rotation around its axis, the earth's fluctuating gravitational pull on the spaceship, etc. In order then to solve this problem, one would have to somehow figure out the equation that would describe the exact trajectory of the spaceship during its flight time. Without any room for trials and with no room for error this problem was presented to Katherine Johnson and she solved it by working backwards. Initially she asked her superiors where on the surface of the earth did they want this spaceship to land. After she found out the ideal recovery points for the spaceship, she then worked out the equations that would give her the answer she needed. Her work is described in detail in "Determination of azimuth angle at burnout for placing a satellite over a selected earth position", a paper she co-authored with T. H. Skopinski.

In her own words: "The early trajectory was a parabola, and it was easy to predict where it would be at any point. Early on, when they said they wanted the capsule to come down at a certain place, they were trying to compute when it should start. I said, 'Let me do it. You tell me when you want it and where you want it to land, and I'll do it backwards and tell you when to take off.' That was my forte."

Katherine has since received several awards for her professional accomplishments. In 1999, Katherine was named West Virginia State College Outstanding Alumnus of the Year. On November 24, 2015 President Barack Obama presented Katherine with the Presidential Medal of Freedom; Katherine was one of the 17 Americans who were awarded this metal. In 2016, Johnson was included in the list of "100 Women", BBC's list of 100 most influential women worldwide and in the same year NASA named a3,700 m^2 building Katherine G. Johnson Computational Research Facility, thus effectively dedicating it to Katherine.

SHAFI GOLDWASSER

Shafrira (Shafi) Goldwasser is an American-Israeli computer scientist, born in New York City in 1958. Her studies involve mathematics and science at Carnegie Mellon University, and also a master’s degree and PhD in computer science at the University of California, Berkeley. Afterwards, she joined MIT and became the first to acquire the RSA Professorship. She also became a professor at the Weizmann Institute of Science and is a member of the Theory of Computation group at MIT Computer Science and Artificial Intelligence Laboratory, where she teaches engineering and computer science. Goldwasser has twice won the Gödel Prize in theoretical computer science. Amongst other awards, she was awarded the 2012 Turing Award along with Silvio Micali for their work in the field of cryptography.

As it is widely known, cryptography has been, throughout the course of time, a capital way of secret communication, influencing even the outcome of wars! Today, however, its role is even more paramount, since the digital reality in which we live imposes delicate and complex challenges to cryptography, which require shifting the traditional goals of cryptography, namely secure and authenticated communication, and move towards systems that are simultaneously highly efficient, highly secure and highly functional.

Shafi Goldwasser has made fundamental contributions to this kind of cryptography, computational complexity, computational number theory and probabilistic algorithms. Her innovative approaches have had an impact on everything within her scientific field, and they ensure security in many aspects of the digital age. It has been said for her (and her co-worker Micali): “they created mathematical structures that turned cryptography from an art into a science”.

More specifically, what Goldwasser is mainly known for is her participation in the development of Probabilistic Encryption, which won her the Turing award, and is what I am going to present.

But, let’s start from the beginning. The general idea behind encryption schemes is how to find a way to ensure secure and private communication through an insecure channel, which may or may not be tapped by an attacker. The information wanted to be communicated is called the plaintext, which must be transformed (encrypted), in order to be illegible to an adversary, but legible to the intended receiver, and the form which is produced is called the ciphertext. The latter must have some valuable information, known only to him, which would enable him to decrypt the message and obtain the original plaintext. That special information is called the key. Three algorithms form an encryption scheme: the encryption algorithm, which converts the plaintext into a ciphertext, the decryption algorithm, which does the opposite procedure, and the key generator, which creates pairs of keys, the encryption and decryption keys. The key generator is considered to be a probabilistic algorithm.

The first encryption schemes were symmetrical, which meant that the same key was used for both encryption and decryption, thus it was better kept private. In the 1970s a new type of encryption was introduced by Diffie and Hellman, called public-key or asymmetric encryption, where there are two different keys for encryption and decryption, and, given the former, it must be impossible to find the latter. The encryption and decryption keys are otherwise called public and private keys, respectively.

The first public-key encryption schemes were based on trapdoor functions, and were what we call deterministic algorithms. These are algorithms that are easy to compute but hard to invert, unless some information is known, which is called the trapdoor. So, while everyone can encrypt a message, only the legal receiver can decrypt them using the trapdoor, which works as the decryption key.

However, Goldwasser and Micali, acknowledging two main drawbacks in encryption schemes based on trapdoor functions, inspired them to develop probabilistic public-key encryption schemes, where they substituted trapdoor functions with trapdoor predicates: A predicate B is trapdoor and unapproximable if anyone can select an x such that B(x) = 0 or y such that B(y) = 1, but only those who know the trapdoor information can, given z, compute the value of B(z). Goldwasser used the predicate “is quadratic residue modulo composite n”.

Her scheme uses bitwise encryption, meaning it depends on a sequence of random computer bits. However, messages are always uniquely decryptable. Two properties of Goldwasser’s probabilistic encryption scheme are: Firstly, decoding of a message is easy for the legal recipient, who knows the trapdoor information, but provably hard for an attacker. Secondly, no information about the plaintext could be obtained from the ciphertext by an attacker.

Goldwasser has also contributed to computer science in various different ways. She is the co-inventor of zero-knowledge proofs, which probabilistically and interactively demonstrate the validity of an assertion without conveying any additional knowledge, and are a key tool in the design of cryptographic protocols. Her work in complexity theory includes the classification of approximation problems, showing that some problems in NP remain hard even when only an approximate solution is needed.

MARYAM MIRZAKHANI

Dr. Maryam Mirzakhani was born in 1977 in Tehran, Iran. She was just 3 years old when the 8-year Iran-Iraq war began. Admitting that these were hard times, she considered herself lucky that the situation in her country had stabilized by the time she reached adolescence.

As a child, she dreamt of becoming a writer and spent her free time reading anything she could find. However, she credited her older brother as the person who piqued her interest in science in general, as he used to come home from school and talk over what he had learned with her.

During her middle school years, she attended the Farzanegan School for girls in Tehran. While there, she didn't do particularly well at mathematics. Surprisingly, a teacher discouraged her interest in the subject by telling her that she wasn't particularly good at it because she wasn't at the top of her class. As a result, she lost interest. Fortunately, a subsequent teacher encouraged her and this led to her showing great talent in mathematics during her remaining high school education. Mirzakhani once indicated that she never thought she would pursue mathematics before her last year in high school. She once happened to find a copy of six Mathematical Olympiad problems and managed to solve three of them. Encouraged by this, she asked her high school principle if she could take part in the Iranian Mathematical Olympiad team even though no girls had ever done so before. The principle agreed and Mirzakhani went on to become an outstanding competitor, earning a gold medal both years she competed along with a perfect score the second year.

By 1999, she had earned her BSc degree from the Sharif University of Technology. During her time there, she had befriended inspiring mathematicians and found that the more time she spent on the subject, the more excited she became. She then went to the US for graduate work, earning her Ph.D. in 2004 from Harvard University. While attending Harvard, she was known for her persistent questioning despite not being a native English speaker and for taking her class notes in Persian. After graduating from Harvard, she became a Clay Mathematics Research Fellow and a professor at Princeton University.

Mirzakhani once said that mathematics is fun, and it made her feel like a detective; that it's like solving a puzzle or connecting the dots in a detective case. This great love of mathematics together with her brilliant mind is what brought Dr. Mirzakhani to the pinnacle of her field. In 2014, she received her most internationally recognized achievement at 37 years old by becoming the first woman ever to be honored with the Field's Medal since the award's establishment nearly eighty years ago.

Some of Dr. Mirzakhani's main contributions are in the dynamics and geometry of Riemann surfaces. Riemann surfaces are one dimensional complex manifolds. In differential geometry, a complex manifold is a manifold with an atlas of charts to the open unit disk in C^n. A complex number is a number that can be expressed in the form $a + bi$, where a and b are real numbers, and i is a solution of the equation $x^2 = -1$, which is called an imaginary number because there is no real number that satisfies this equation. For the complex number $a + bi$, a is called the *real part*, and b is called the *imaginary part*. One of her contributions was determining a general formula for the number of closed geodesic curves in a Riemann surface that has a very large genus. A geodesic curve is the shortest path between two points on a Riemann surface. A closed geodesic curve is when the geodesic curve forms a closed loop. A genus is an integer representing the maximum number of cuttings along non-intersecting closed simple curves without rendering the resultant manifold disconnected. Generally, the most common concept is the number of "holes" the surface has.

Farb said that solving each of the problems that Dr. Mirzakhani solved "would have been an event, and connecting them would have been an event". Mirzakhani did both.

Dr. Maryam Mirzakhani, who passed away on July 14, 2017 due to breast cancer, left a lasting legacy which will always serve as an important reminder that the language of mathematics is a universal language, indiscriminate of where someone is from, what language they speak or their gender. Although Dr. Mirzakhani once stated that she didn't believe she'd made a very big contribution, most would disagree. Indeed, she led an intriguing, albeit short, life which is marked by a great legacy of accomplishments and achieved phenomenal heights of renown as a mathematician.

CONCLUSION

Upon being awarded the Field's Medal, Dr. Mirzakhani stated that she would be happy if her winning encourages young female scientists and mathematicians so that many more women will aspire to win this kind of award in coming years. Perhaps her greatest legacy will not only be her brilliant contributions to mathematics, but the inspiration that young women and girls like us from around the world draw from her trailblazing success.

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GOLDEN RATIO

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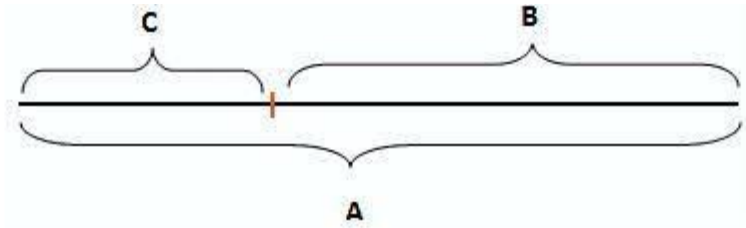
ABSTRACT

The science of Mathematics with its numerous theorems and propositions plays an important role in several areas of our lives. This is true, despite the fact that we may not always be able to recognize the application of mathematical results every single time we encounter or use them. In order to justify this argument we have decided to focus on a particular mathematical discovery - namely the golden ratio. The golden ratio, denoted by mathematicians with the symbol φ, corresponds to a particular irrational number. What makes φ unique and deserving of our attention is the large number of instances where we come across it. Indeed, we will demonstrate how φ and its properties have influenced almost every aspect of human life and we will present occasions where the number φ is found in natural circumstances.

Greek letters are very frequently used to denote and communicate concepts that are often complicated and difficult to understand. They represent notation that is well recognized by mathematicians, engineers and scientists in all fields, internationally. One such number is the number represented by the Greek letter φ, which is an irrational number with a value of 1.6180339887… Simply looked at as a number, φ does not seem to present any interest at all. As an irrational number, it is defined as the ratio of lengths of two line segments, which share no "measure" in common, meaning that the ratio of their lengths cannot be expressed as a ratio of whole numbers. As such, it may appear that φ has no interest from a geometry point of view, either. Nevertheless, φ has two mathematical properties, that make it unique and seem to add 'magic' to it. These two properties are the following:

- If you square φ you get a number exactly 1 greater than itself: 2.6180339887… (or $\varphi^2 = \varphi + 1$)
- If you divide φ into 1 to get its reciprocal, you get a number exactly one less than itself: 0.6180339887… (or $1/\varphi = \varphi - 1$)

Furthermore, from a geometry point of view, φ is a ratio of the length of two line segments, defined as follows. Consider a straight line segment A cut in two segments B and C, as shown in the figure below, so that the ratio of the length of segment B to that of segment C is the same as the ratio of A to B (mean and extreme ratio). This ratio is equal to φ. These properties make φ a really unique number, which excited many scientists from the ancient times.



ORIGINS OF THE GOLDEN RATIO

Pythagoreans, Pythagoras' students and followers in the 5th century B.C., believed that whole numbers and their ratios were the essence of all things. In their approach they thought that all numbers could be expressed as a ratio of integers. It was them, however, who through their study of numbers discovered the irrational numbers. As such, Pythagoreans' discovery of the irrational numbers may be considered as the beginning of the "existence" of φ, even though φ was not formally discovered yet. Furthermore, the five-pointed star which is the symbol of the Pythagoreans, also used as a sign of salutation by them, encompasses the essence of φ. The five-pointed star and the pentagram (the star encased in a circle) appear in findings dating back to even the Chalcolithic period (4500 – 3100 BC) and have been used as a symbol by many ancient civilizations in Egypt, Mesopotamia and elsewhere, as well as in contemporary cultures. Pythagoreans called the pentagram υγίεια (narrowly translated as health) in the sense of wholeness or divine blessing. The pentagram includes ten golden triangles, in all of which the ratio of the longer side to shorter side is φ.

The proportion represented by φ must have been studied by Phidias in the 5th century B.C., as he seems to have applied it to the design of the Parthenon's sculptures. Explicit reference to this proportion is also made in the works by Plato (428 – 347 BC) and Euclid (365 – 300 BC), who were both influenced by the work of the Pythagoreans. Plato, in his philosophical dialogue "Timaeus" presents a distinction between the physical world and the eternal one, and an explanation of the creation of the universe and its properties. He further explains how the soul of the world was created, makes assumptions on the composition of the four elements (earth, water, air, fire), presents the fifth element representing the Universe, and links each element to a certain Platonic solid. The Platonic Solids are convex polyhedra with equivalent faces composed of congruent convex regular polygons. The five regular solids (where "regular" means all sides are equal, all angles are the same and all the faces are identical) are called the five Platonic solids. They are the tetrahedron, cube (or hexahedron), octahedron, dodecahedron and icosahedron. Plato gives special praise to the dodecahedron, symbol of the Universe, which is "superficially beautiful and rational, but it contains hidden the Golden Ratio and its imperfection". It is however unclear who originally discovered the number φ, which is also known as the Phi ratio and the Golden Ratio. Other names include the golden mean or golden section (Latin: sectio aurea) extreme and mean ratio, medial section, divine proportion, divine section (Latin: sectio divina), golden proportion, golden cut and golden number. The first clear definition of the Golden Ratio was given around 300 B.C by Euclid of Alexandria who is considered the father of geometry. In 'Euclid II.11' he defined a proportion derived from a simple division of a line into what he called its "extreme and mean ratio", as mentioned above. His analysis is the basis for the development of the Fibonacci sequence.

The Fibonacci sequence is another way to derive Phi mathematically. It is formed as a series of numbers derived by starting with 0 and adding 1 to get 1. Then the process is repeated by adding the sum of the last two numbers as the next number in the series: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 and so on. The relationship to the Golden Ratio is found by dividing each number by the one before it. The further you move in the series, the closer the result gets to the number φ. For example:

1/1=1	2/1=2	3/2=1.5	5/3=1.666
13/8=1.625	21/13=1.615	...	

This sequence was known as early as the 6th century AD by Indian mathematicians but it was Fibonacci that introduced it to the west.

THE GOLDEN RATIO IN ART AND DESIGN

From Renaissance till today, artists have recognized the ability of Phi to give a sense of balance and harmony and have used it in some of the world's greatest works of art. According to some researchers and analysts, the golden proportion is considered to be the most aesthetically pleasing and it is said to be found in the human body as well as in animals and in the growth pattern of many plants. As such it could be considered natural that artists follow it in their work. Other researchers and analysts, however, challenge the importance of the Golden Ratio in aesthetics and the extent to which it has been intentionally used in the arts.

As an example, the drawing of the Vitruvian Man, by Leonardo da Vinci is perceived to be a picture of perfection. The standing image of the man is inscribed in a square while the image drawn with legs and arms spread apart is inscribed in a circle. It is considered to be a characteristic example of how art and science are compound and exemplifies da Vinci's interest in proportion. Actually, the Vitruvian Man demonstrates a broad variety of proportions in the human body. Some of these proportions coincide with the Golden Ratio. Another piece of work by da Vinci, in which the artist seems to have used the Golden Ratio is "Mona Lisa". It is said that if you draw a rectangle around the face of Mona Lisa, the ratio of the height to the width of the rectangle is equal to the Golden Ratio. However, it is not said where exactly should the rectangle be drawn. In his painting "The Last Supper", da Vinci used the Golden Section in framing the whole picture, meaning in positioning the table and Jesus within the frame, as well as in many details of the painting. Evidently, da Vinci used the Golden Ratio extensively in his work. However, whether he did it consciously or not is subject of heated debate.

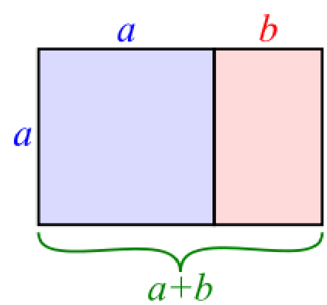
Salvadore Dali is another artist who is considered to have used the Divine Proportion in his work. In his painting 'The Sacrament of the Last Supper', Dali used a golden rectangle as the frame of the painting and the golden proportion in the way the table was placed in the painting and the way the two disciples were placed at Christ's side. Moreover, the windows are formed by a dodecahedron, one of the Platonic Solids.

In sculpture, the statue of Venus de Milo is found to be strictly in the Golden Proportion. The statue Doryphoros by Polykleitos, which is one of the best known Greek sculptures of classical antiquity, may be considered to be divided in three zones. The first one from the top till the right nipple, the second from the right nipple to the knee and the third from the right knee to the right big toe. The second zone is given a length of 1.61803 while the other two are given a length of 1. Some claim that these measurements are not accurate and that the left side of Doryphoros does not have the same proportions.

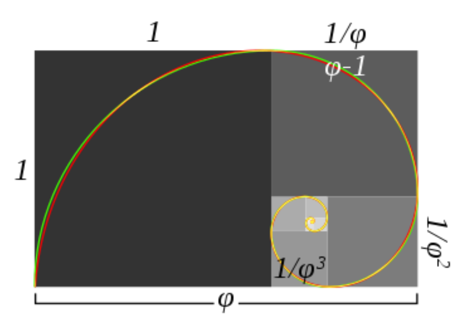
THE GOLDEN RATIO IN NATURE

The golden ratio is observed throughout the shape and structure of the human body. The ratio from head to toe over head to fingertip, equals the ration from head to fingertip over fingertip to toe, according to the Golden proportion. Similarly, the arm consists of three parts, and the finger consists of three phalanxes, each relating to the other according to the rule of Golden Proportion. Furthermore, the human body has one trunk, one head and one heart. Some parts such as arms, legs and eyes are in pair. Arms and legs are made of three parts. There are five fingers on each hand and foot, as well as five openings on the face. One, two, three and five are numbers of the Fibonacci series. The Golden Proportion is claimed to be the reason why we consider some people to be more beautiful than others. It is because the proportions of various features of their face are close to the Golden Ratio. However, many disagree with this theory and say that when computers where used to change face proportions away from average, the faces that were created where preferred by some people. Only very few facial proportions are shown to play a role on the perception of attractiveness.
Besides humans, examples of the Golden Section are found on plants and animals. The Golden Spiral or Fibonacci Spiral is observed in various cactuses, and shell growths. Several plants produce new branches following the Fibonacci numbers. The star fish has a pentagram-like structure. The Golden Proportion is found in many animals, fishes, birds, insects. Evidently, the golden ratio is found everywhere in nature. Furthermore, the Golden Proportion is found elsewhere in the Universe, such as in the formation of the Galaxy and the orbital distances of planets from the earth.

LINE SEGMENTS IN THE GOLDEN RATIO



A golden rectangle (in pink) with longer side ***a*** and shorter side ***b***, when placed adjacent to a square with sides of length ***a***, will produce a similar golden rectangle with longer side ***a* + *b*** and shorter side ***a***. This golden ratio participates in the design of the spiral met in architecture but its origins can be found in nature. The spiral is made from quarter-circles tangent to the interior of each square. The length of the side of one square divided by that of the next smaller square is the golden ratio. The number *φ* turns up frequently in geometry, particularly in figures with pentagonal symmetry. The length of a regular pentagon's diagonal is *φ* times its side.



Many of the proportions of the Parthenon allegedly exhibit the golden ratio. The Parthenon's façade as well as elements of its façade and elsewhere are said by some to be circumscribed by golden rectangles. Other scholars deny that the Greeks had any aesthetic association with golden ratio. For example, Midhat J. Gazalé claims that it was not until Euclid that the golden ratio's mathematical properties were studied. In the *Elements* (308 BC) the Greek mathematician merely regarded that number as an interesting irrational number.

Elsewhere, the Great Mosque of Kairouan, the most important mosque in Tunisia, situated in the UNESCO World Heritage town of Kairouan, reveals a repeated application of the golden ratio throughout its design, according to Boussora and Mazouz. They found ratios close to the golden ratio in the overall proportion of the plan and in the dimensioning of the prayer space, the court and the minaret.
Besides, principal authorities on the history of Egyptian architecture have argued that the Egyptians were well acquainted with the golden ratio and that it is part of mathematics of the Pyramids. Historians of science have always debated whether the Egyptians had any such knowledge or not, contending rather that its appearance in an Egyptian building is the result of chance. Pyramids specialist, John Taylor claimed that, in the Great Pyramid of Giza, the ratio of the length of the face (the slope height), inclined at an angle θ to the ground, to half the length of the side of the square base, equivalent to the secant of the angle θ represents the golden ratio. The above two lengths were about 186.4 and 115.2 meters respectively. The ratio of these lengths is the golden ratio. Similarly, Howard Vyse, according to Matila Ghyka, reported the Great Pyramid' s height 148.2 m, and half-base 116.4 m, yielding 1.6189, almost the Phi.
In fact, it is not before the middle of the nineteenth century that the Golden Section enters architectural theory. In the time of the beginning of Historicism and of the great scientific discoveries and theories, Adolf Zeising (1810-76) began his researches on proportions in nature and art. The Swiss architect Le Corbusier, explicitly used the golden ratio in his Modulor system for the scale of architectural proportion. He considered this system as a natural continuation of Leonardo da Vinci's "Vitruvian Man", the work of Leon Battista Alberti, and others who tried to improve the appearance and function of architecture by using the proportions of human body. Le Corbusier's 1927 Villa Stein in Garches applied the Modulor system. The villa's rectangular ground plan, elevation, and inner structure closely approximate golden rectangles.
Another Swiss architect, Mario Botta, designed several private buildings in Switzerland which are composed of squares and circles, cubes and cylinders frequently using the golden section as base proportion among the dimensions of the geometrical shapes. After a long and comprehensive study however, Marcus Frings concludes that for a long time the Golden Section does not occur in architectural theory. It first appears in the nineteenth century, through Zeising and Fechner, and gains prominence in the 1920s and 1930s through the work of Neufert and Le Corbusier. In any case, these analysts claim that the Golden Section plays a role in the drawings of these architectural theorists. Before the 19th century, however, the Golden Section is simply absent in written architectural theory.

GOLDEN RATIO IN MUSIC

Moving our focus to music we will notice that Fibonacci and phi relationships are often found in the timing of musical compositions. As an example, the climax of songs is often found at roughly the phi point (61.8%) of the song, as opposed to the middle or end of the song. In a 32 bar song, this would occur in the 20th bar.
John Putz analyzed W.A. Mozart's piano sonatas to verify if, as already mentioned by others, these works do reflect the golden ratio. He analyzed the ratio of the duration of Exposition in which the musical theme is introduced to the duration of the Development and Recapitulation in which the theme is developed and revisited. He concluded that in most cases the structure of Mozart's sonatas reflect the Phi for the ratio of a to a + b. Other music theorists who analyzed

Bela Bartok's works, one of the most important composers of the 20th century, have found out that they are created according to two opposing systems, that of the golden ratio and the acoustic scale.

Besides, Erik Satie, a French composer used the golden ratio in several of his pieces, including *Sonneries de la Rose+Croix*. The golden ratio is also met in the structure of the sections in the music of Debussy's *Reflections in Water*, from *Images* (1st series, 1905), in which the sequence of keys is marked out by the intervals 34, 21, 13 and 8, and the main climax sits at the phi position. Musicologist Roy Howat has observed that the formal boundaries of *La Mer* correspond exactly to the golden section.

The golden ratio does not only influence the structure of some musical masterpieces but is also taken into account in the manufacturing of some musical instruments. Fibonacci and phi are used in the design of violins and even in the design of high quality speaker wire. The percussion and drums producer company Pearl Drums positions the air vents on its Masters Premium models based on the golden ratio. The company claims that this arrangement improves bass response and has applied for a patent on this innovation.

Furthermore, it seems that the golden ratio is an inherent feature of western music itself. Musical scales are related to Fibonacci numbers whose ratios tend to Phi:

- There are 13 notes in the span of any note through its octave (count the black and white keys from C to next C).
- A scale is composed of 8 notes (take only the white keys), of which the 5th and 3rd notes create the basic foundation of all chords, and are based on a tone which are combination of 2 steps and 1 step from the root tone, that is the 1st note of the scale.

In a scale, the dominant note is the 5th note of the major scale, which is also the 8th note of all 13 notes that comprise the octave. This provides an additional example of Fibonacci numbers in key musical relations. Interestingly, 8/13 is 0.61538, which approximates phi. Looking now deeper into the musical frequencies, we realize that they are based on Fibonacci ratios and Phi. Notes in the scale of western music are based on natural harmonics that are created by ratios of frequencies. Ratios found in the first seven numbers of the Fibonacci series (0, 1, 1, 2, 3, 5, 8) are related to key frequencies of musical notes.

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MATHEMATICS IN COMPUTER GAMES

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ABSTRACT

In computer games, Mathematics is used to make the game work, without it the games we have now would just not exist. No one even suspects what magic lies behind the colourful screens. In fact, one of the most well-known theorems in the world is used to calculate the distance from objects – Pythagoras' Theorem. It helps to calculate the hypotenuse of triangles, which can be used to move the characters. As well as that, trigonometry is used to calculate the angle and then the bearing from the objects, so that sprites can later on use it to turn to face the aforementioned object. In addition, moving a sprite from one place to another diagonally is not just one simple line because there is no function that will move it in such a way. Therefore, the program calculates the gradient at which the character needs to move. Due to the ability of computers to process thousands of instructions in seconds, it will repeatedly print a new image, giving a viewer the perception that the sprite is moving.

INTRODUCTION

First of all, what are computer games? They are made up of lines of code. For example, Minecraft requires 4,815,162,342 lines of code.1 More specifically, all the colourful games and animations that are seen every day are ultimately traceable back just ones and zeros from binary code. However, in my simulation, I would like to show how much mathematical based is our world of computer games as they all use the Pythagoras theorem, trigonometry, calculus and many other mathematical equations.

To show the usage of Mathematics in computer games, I have written a simple code which prints two characters, Mr.Redsquare and Mr.Bluesquare to the screen. After a series of calculations, Mr.Bluesquare will obtain the exact position of Mr.Redsquare and will move to that exact position, no matter where Mr.Redsquare appears on the screen.

STAGE 1

In the very first step all that the code does is ask for inputs to begin the game. After that, it imports the assets of pygame to allow an image to appear in a separate screen. As well as that, the size and the title of the game screen is identified which then opens a 700px by 700px screen. Finally, the images of the players are uploaded to the programme and set to a variable which will be easier to use later in the game.

[1] Pecany, 2018

```
import sys

print("  ***** Demonstration Game *****  ")
print(" ")
question = raw_input("Write 'start' to begin the game? ")
print(" ")
cancel = "run"

while 1:

    if question != "start":
        answer = raw_input("Are you sure you want to quit? ")
        print(" ")
        if answer == "yes":
            pygame.quit()

        else:
            question = "start"
            answer = "no"

    print(" ")
    print(" ")

    import pygame

    pygame.init()
    width = 700;
    height = 700
    pygame.display.set_caption("Demonstration game")
    screen = pygame.display.set_mode((width, height))
    background = pygame.display.set_mode()

    red_square = pygame.image.load("player.bmp").convert()
    blue_square = pygame.image.load("player2.bmp").convert_alpha(background)
```

In the second part, I have imported the random function to enable my code to choose a random value. As this simulation must show that at any position on the screen, Mr.Bluesquare will be able to identify the exact coordinates of Mr.Redsquare, therefore both characters must be printed in a random position on the screen every time. Giving the computer a range of values between 0 and 600, it will randomly choose 4 values and give each sprite an x and y axis coordinate. The last few lines paint the screen white and print the characters to the game board.

```
import random
x = int(random.randint(0,600));
y = int(random.randint(0,600));
print("The x coordinate of Mr.RedSquare = {} " .format(x))
print(" ")
print("The y coordinate of Mr.RedSquare = {}" .format(y))
print(" ")

x2 = int(random.randint(0,600));
y2 = int(random.randint(0,600));
print("The x coordinate of Mr.BlueSquare = {} " .format(x2))
print(" ")
print("The y coordinate of Mr.BlueSquare = {} " .format(y2))
print(" ")

screen.fill((255,255,255))

screen.blit(red_square,(x,y))
screen.blit(blue_square,(x2,y2))

pygame.display.update()

print(" ")
print(" ")
```

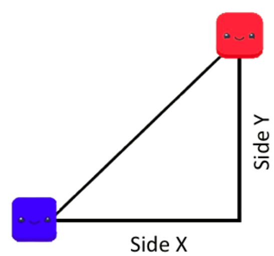
STAGE 2

At stage 2, the code is 61 lines in and due to the need for mathematical functions, the asset, 'math', is imported to enable trigonometry and many other mathematical calculations.

Between two objects a right-angled triangle can always be formed, as a result of this, now we can calculate Side x and Side y of this triangle which will be extremely helpful in our next stage. In addition, due to combinatorics we can identify that there are only four possible ways the triangle can be orientated to show the path of Mr.Bluesquare, as well as showing how many varieties of the same code need to be written for the stimulation to work every time. There will always be two positions at which Mr.Bluesquare will be higher than Mr.Redsquare and two position when he will be below. Similar with left and right, due to this we take the two values and multiply them together to get a total value of four possible ways Mr.Bluesquare can move.

As we already know the coordinates of our characters, all that is left to find out is who is above or on the left of the characters, and to then subtract the coordinate values to obtain the lengths of the right angles triangle formed between the two characters. An example of one of the triangle is show bellow.

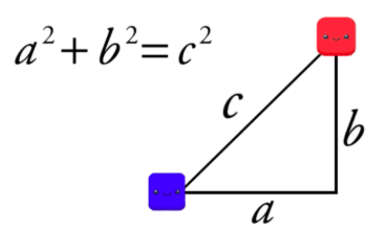
```
import math

stage1 = raw_input("          Stage 1 ")
print(" ")
if stage1 == "":
    sidex = x2 - x
    if sidex > 0:
        print("Side x is = {}" .format(sidex))

    else:
        sidex = x - x2
        print("Side x is = {}" .format(sidex))

print(" ")

sidey = y2 - y
if sidey > 0:
    print("Side y is = {}" .format(sidey))

else:
    sidey = y - y2
    print("Side y is = {}" .format(sidey))

print(" ")
print(" ")
```



STAGE 3

As a result of calculating the two sides of the triangle, we are now able to use the Pythagoras theorem to calculate the hypotenuse or the distance between the two sprites. However, there was a minor problem as there is no function in the math asset which will square a number, to solve this problem I rewrote this equation in another form → (a × a) × (b × b) = c × c. After calculating c^2 I used one of the function from the assets to square root the number to store the hypotenuse value in a variable.

```
stage2 = raw_input("          Stage 2 ")
print(" ")
if stage2 == "":
    hypsqt = (sidex * sidex) + (sidey * sidey )
    hyp = math.sqrt(hypsqt)
    print("The hypoteneus = {}" .format(hyp))


print(" ")
print(" ")
```

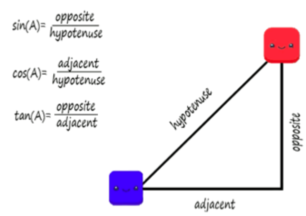


STAGE 4

In stage four, the code calculates the angle between the hypotenuse and the adjacent using trigonometry, more specifically using the tangent rule.

First of all, the program will figure out how the triangle is orientated and then according to that will divide the sides. After obtaining tan(A) using inverse tangent, the tan(A) will be converted into an angle.

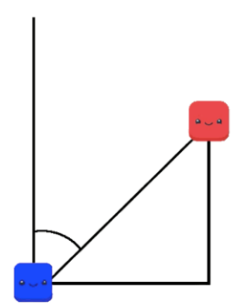
```
stage3 = raw_input("          Stage3 ")
print(" ")
if stage3 == "":
    if y2 > y:
        if x2 > x:
            preangle = float(sidey) / float(sidex)
        elif x2 < x:
            preangle = float(sidey) / float(sidex)
    elif y2 < y:
        if x2 > x:
            preangle = float(sidex) / float(sidey)
        elif x2 < x:
            preangle = float(sidex) / float(sidey)

angle = math.degrees(math.atan(preangle))
print("The angle = {}" .format(angle))
print(" ")
print(" ")
```



STAGE 5

In most games, it is vital for enemies to turn to face where they are going, therefore, using bearing and the angle we calculated in the previous stage will help to calculate the angle at which Mr.Bluesquare needs to turn if he is already facing north. Again, according to the triangle the variable bearing will either be the angle, 360 + angle or 180 ± angle.

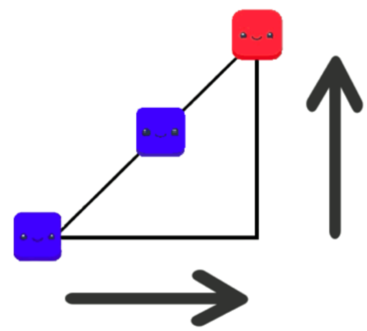
```
stage4 = raw_input("          Stage 4 ")
print(" ")
if stage4 == "":
    if y2 > y:
        if x2 > x:
            bearing = angle

        if x2 < x:
            bearing = 360 - angle

    if y2 < y:
        if x2 > x:
            bearing = 180 - angle

        if x2 < x:
            bearing = 180 + angle

print("The bearing = {}" .format(bearing))

print(" ")
print(" ")
```



STAGE 6

Last but not least, the movement of Mr.Bluesquare. This was also a challenge as there is no ready code which will in a few lines tell a character to move in a diagonal at a particular speed. Instead of that, I calculated the gradient of the hypotenuses by dividing side Y by side X. Then, depending on the orientation of the triangle, the program will either ± the gradient from the y coordinate and ± 1 to the x coordinate. Due to computers ability to quickly calculate this and keep reprinting Mr.Bluesquare in its new position, it looks as if he is moving towards Mr.Redsquare.

```
stage5 = raw_input("          Stage 5 ")
print(" ")

gradient = float(sidey) / float(sidex)
print("The gradient = {}" .format(gradient))
print(" ")

if stage5 == "":
    while x != x2 and y != y2 :
        if x2 > x:
            if y2 > y:
                x2 -= 1
                y2 -= gradient
            elif y2 < y:
                x2 -= 1
                y2 += gradient
        elif x2 < x:
            if y2 > y:
                x2 += 1
                y2 -= gradient
            elif y2 < y:
                x2 += 1
                y2 += gradient

        screen.fill((255,255,255))
        screen.blit(red_square, (x,y))
        screen.blit(blue_square, (x2,y2))
        pygame.display.update()
        print("X = {}          Y = {}" .format(x2,int(y2)))
```

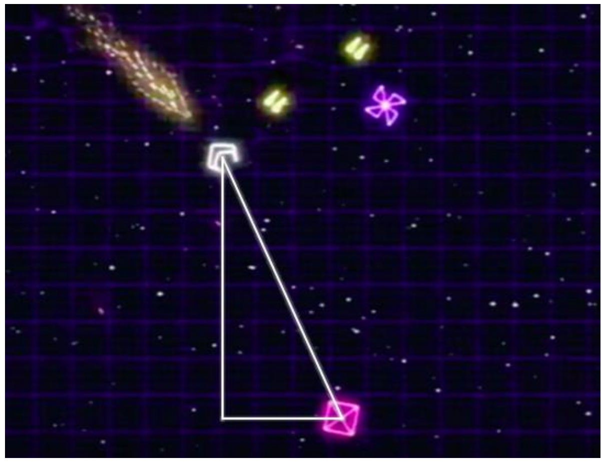


The last part of my code simply askes if you would like to continue and start the stimulation again or end it.

```
    print(" ")
    print(" ")

    print("You Won!!!")
    print(" ")

    finalQ = raw_input("Would you like to start again? ")

    print(" ")
    print(" ")
    print(" ")

    if finalQ != "yes":
        pygame.quit()
        sys.exit()

pygame.quit()
```

CONCLUSION

I have found the game Geometry Wars which may use a similar program to mine as the enemy (pink cube) moves directly towards the main character to attack it.



Overall, this entails only the basics of how Mathematics is incorporated into computer games. There are many more other equations which can be used such as the parabola equation to make a character jump. This proves my point about Mathematics being everywhere, even in computer games as every year games become even more realistic as more mathematical equations are added.

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MATHS OR LUCK?

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ABSTRACT

In this paper, we will be explaining how one can use mathematics in order to increase the probabilities of winning at traditional board games. The best strategies are the ones based on mathematics and we are going to prove this by showing how the probabilities of victory can be drastically increased using such methods. We will be using Monte Carlo simulations and other computational algorithms to back up our research and claims. Through the paper, we will show you which regions are the best for controlling in Risk, when the best time is to attack and when to defend and prove it all using mathematics. We will also show you which are the best questions to ask in Guess Who, and prove why they are the most effective. Finally, we will demonstrate the best properties to buy in Monopoly. We aim to change the way you approach board games, not just as games of chance but as games of skill, knowledge and strategy.

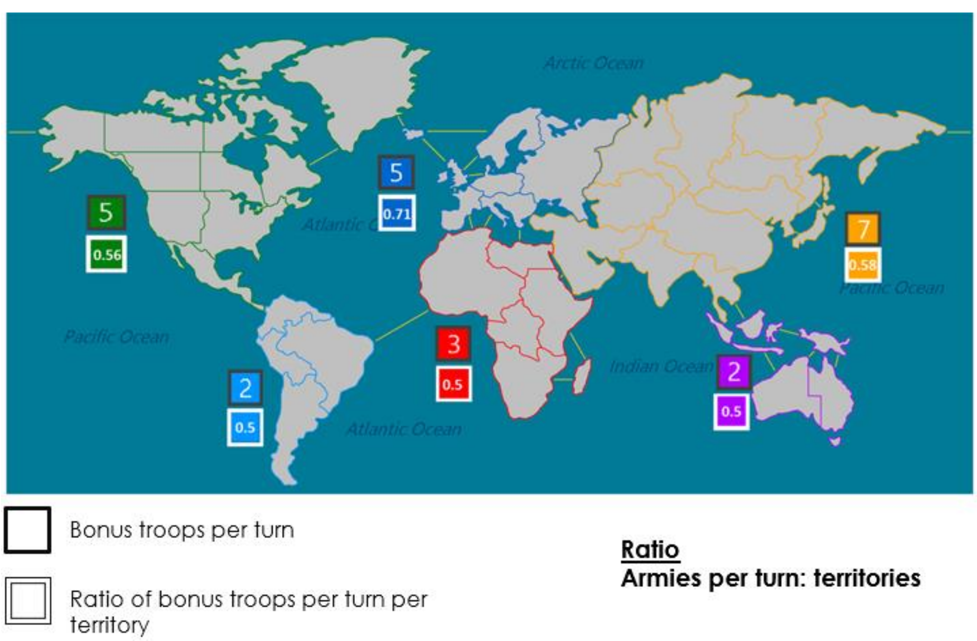
RISK

Risk is a game of world domination, where your objective is to command an army to take over the world. The board is a world map split up into continents, and continents are split up into territories. You win when you control all the continents on the board. At the start of the game, players take turns choosing territories to control. This is done until the map is fully occupied. Players who have troops in a territory can attack adjacent territories on their turn. If you are the sole owner of a continent, you gain bonus troops each turn. Therefore, it is in your interest to control and defend continents in order to build up your army. So, how do you attack? Well, it depends on how many troops you have. As the attacker, you roll as many dice as there are attackers, up to three dice. If you are a defender, you roll as many dice as there are defenders, up to two dice. After the dice are rolled, the highest rolls of the defender's contest with the highest rolls of the attackers. For each roll of the defender which is lower than a roll of the attacker, the defender loses one troop. If the defending territory has 0 troops left, you win the territory. You can attack as many times as you want per turn, as long as you have troops.
In a common 5 player game of risk, each player starts the game with 25 units and every player selects territories one after the other until all territories have been selected. This results to dividing the 42 territories in the following way; 3 players get 9 territories and 2 players get 8 players each. Afterwards, the game begins where all players fight amongst themselves to control entire continents as a bonus number of units per round can be very helpful to shift the probabilities of winning towards their favour.

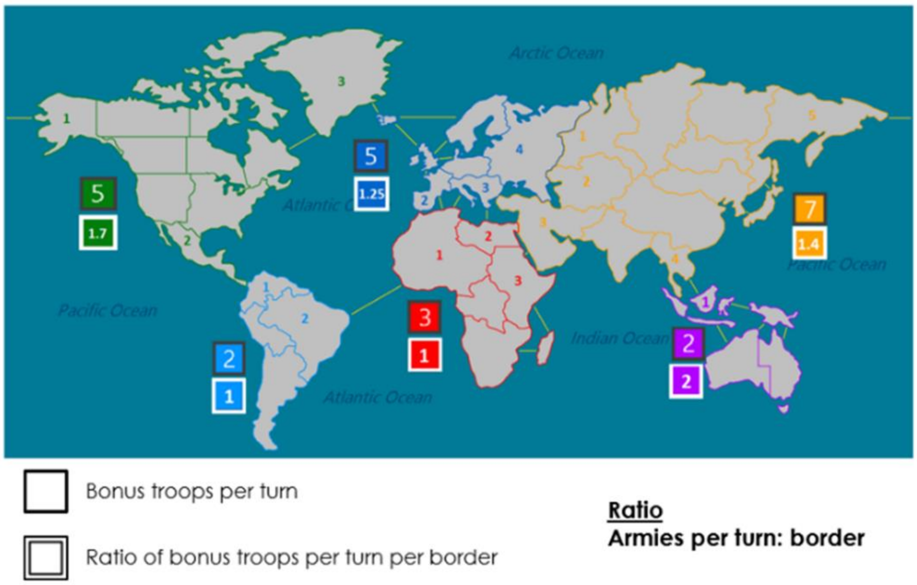
Therefore, a good question to ask would be which continent is the right to go for? Which territories should you focus at controlling initially? To find out, a player should take in consideration the amount of territories in each continent and the respective amount of bonus units each one rewards:

Continent	Number of territories	Number of bonus armies per turn
Africa	6	3
Asia	12	7
Australia	4	2
Europe	7	5
North America	9	5
South America	4	2

To make it much clearer, we must deduce the ratio between the bonus number of units to the number of territories **(Armies per turn: territories)**. As we can see from the graph below Europe seems as the better continent to control as it gives the highest ratio, while smaller continents like South America and Australia are less attractive since they have a reasonably low ratio.



However, there is a more important factor we need to consider… Borders!
A player must not only ensure to obtain a maximum value per territory, but also to minimize the probability that another player can take one territory and strip them out of their continent bonus. Our point is that continents with less borders are much easier to defend and keep under control for a longer period of time, thus enjoying rewards for more rounds. The more territories with borders a continent has, the more vulnerable it is as the player has to split the army in more groups to protect the borders. Therefore, to truly determine which continent is best to control you need to look at the ratio between the bonus number of units to the number of borders and not to the number of territories.



Europe has 4 border territories and a 5 troop per turn bonus, so its bonus troop per border territory ratio is 1.25. Whereas Australia has 1 border territory and a 2 troop per turn bonus, so its bonus troop per border territory ratio is 2.
Australia is obviously the easiest to defend, making it the ideal continent to dominate first so a player can boost their army at a faster rate than their opponents.
Even though, Europe seems as a good choice as risking to control it will give the biggest rewards per territory but its large number of territories and borders, make it very hard to defend.
Since attacking and defending with dice define every interaction in Risk, knowing the ways to use the statistics of battle give a player a distinct advantage when playing. We have done some calculations and here are the results.

Defender Rolls

Attacker Rolls

3 Attacking vs. 2 Defending		
Defence Loses 2	2890/7776	37.17%
Attack Loses 2	2275/7776	29.26%
Attack Loses 1 Defence Loses1	2611/7776	33.58%

3 Attacking vs. 1 Defending		
Defence Loses	855/1296	65.97%
Attack Loses	441/1296	34.03%

2 Attacking vs. 2 Defending		
Defence Loses 2	295/1296	22.76%
Attack Loses 2	581/1296	44.83%
Attack Loses1 Defence Loses	420/1296	32.41%

2 Attacking vs. 1 Defending		
Defence Loses	125/216	57.87%
Attack Loses	91/216	42.13%

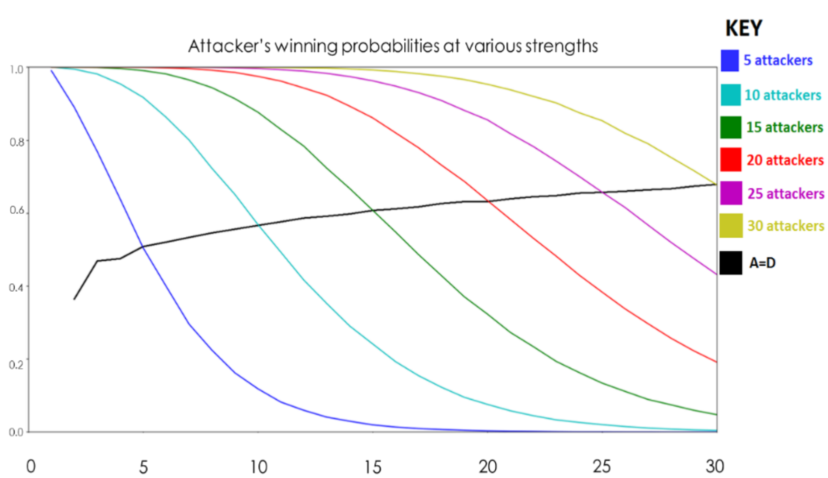
1 Attacking vs. 2 Defending		
Defence Loses	55/216	25.46%
Attack Loses	161/216	74.54%

1 Attacking vs. 1 Defending		
Defence Loses	15/36	41.7%
Attack Loses	21/36	58.33%

For example, when the attacker rolls three dice and the defender two the probability of the defender losing two troops is 37.17%. When the attacker rolls 2 dice and the defender 2 the chances of the defender losing two troops falls to 22.76%.

Generally speaking, whoever rolls more dice has an advantage. When the dice are evenly matched, the defender tends to win. The larger the battle the larger the attackers advantage.

Now let's focus on this graph that was generated by a computer program we wrote. We can see the probabilities of the attacker winning in the y axis and the number of defending troops on the x- axis while having the number of attacking troops displays by each line. For example, the red line shows the probabilities of the attacker winning when the attacker has 20 troops. According to these graph, whenever the attacker has a bigger number of troops than the defender, the chances of him coming out of the battle victorious are with him. However, let's concentrate on the line, A=D. We can see that in this type of situation, when the 2 armies are balanced, at 5 troops each there is a probability of almost exactly 0.5. But as more troops are added equally to each army the probability of the attacker winning increases. This shows that in general the larger the battle, the bigger the probability the attacker has to win.



GUESS WHO

Guess Who is a two-player game where competitors use yes or no questions to isolate a hidden character. The first player to guess their opponents character wins.
We have come up with three different strategies for Guess Who.
The first one is called the “Super narrow question”. A super narrow guessing strategy is where the question applies to only one specific character. E.g. Does your character wear blue glasses? The probability of guessing your opponent’s character on the first guess is 1 out of 24. But it is equally likely that you do not get it right until the final and 24th guess. On average, it will take 12.5 guesses to win.
The second strategy is called “Narrow question”. A narrow question applies to only 5 people. The chance that the answer to a narrow question is YES at the beginning is about 20.83% and if it is, then you have narrowed it down to 5 people and practically won.
On the other hand, if you ask your opponent a broader question that applies to 10 people there is a greater chance that the answer will be yes, 41.67%. But because it is a broader question, the number of characters left standing are 10 as opposed to only five.
The third strategy is the “Super broad question”. The broadest question would apply to exactly half of the characters. Therefore, either the answer is a yes or a no the number of characters that you knock down is the same, 12. Then you can ask another broad question that applies to half of the remaining characters until you are left with 1 option.

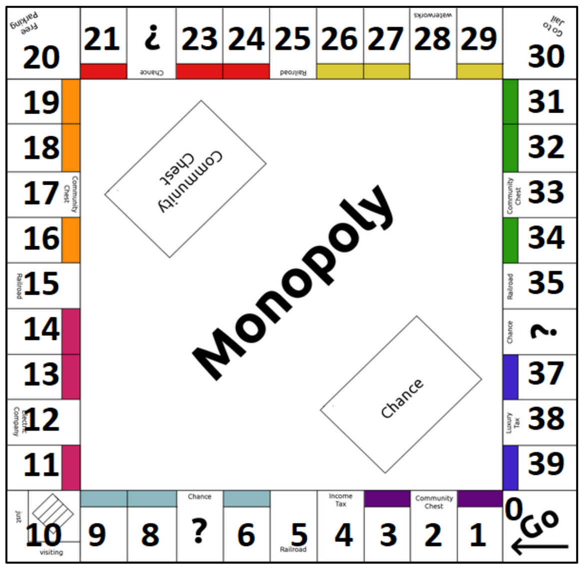
24 → 12 → 6 → 3 → 2 or 1

In total, it will take you 5 or 6 moves.
There are several different ways in which someone can ask a question that applies to exactly half of them.
E.g. does your character have white hair or red hair or wears glasses? This type of question is allowed since the rules state a player must ask a yes or no question.
Then you just have to ask a combination of questions that apply to the remaining half people.
In conclusion, the best strategy is the last one with the super broad questions as on average it takes 5.5 questions to win.
To increase the chances of winning even more the key is to play first one to five wins. ´Guess Who´ calls this the championship mode and some form of their board game even provided pegs to keep track of the number of wins.
The reason that this increases your chances so much is because of the Law of Large Numbers. As the number of trials or observations increases, the actual or observed probability approaches the theoretical or expected probability.
Since a player has the better strategy (i.e. The super broad question) the more games he plays, the more obvious it becomes, and the opponent’s beginners luck will eventually run out.

MONOPOLY

The final game we will investigate is Monopoly. In Monopoly you use two dice to move around the board. When you land on a property, you can buy it for the price listed and after that

whenever someone else lands on that property they have to pay rent. Whenever you pass the starting position of the board, you gain extra money, and if you land on the "GO TO JAIL" square you are immediately taken to the JAIL square. The aim of the game is to lead all of your opponents to bankruptcy, giving you a monopoly over the board.
Buying properties is an essential aspect of the game and knowing what properties to buy can increase your chances of winning. So, which are the best properties in “Monopoly”? For simplicity reasons we will use the board below for our examples referring to each square with its number.

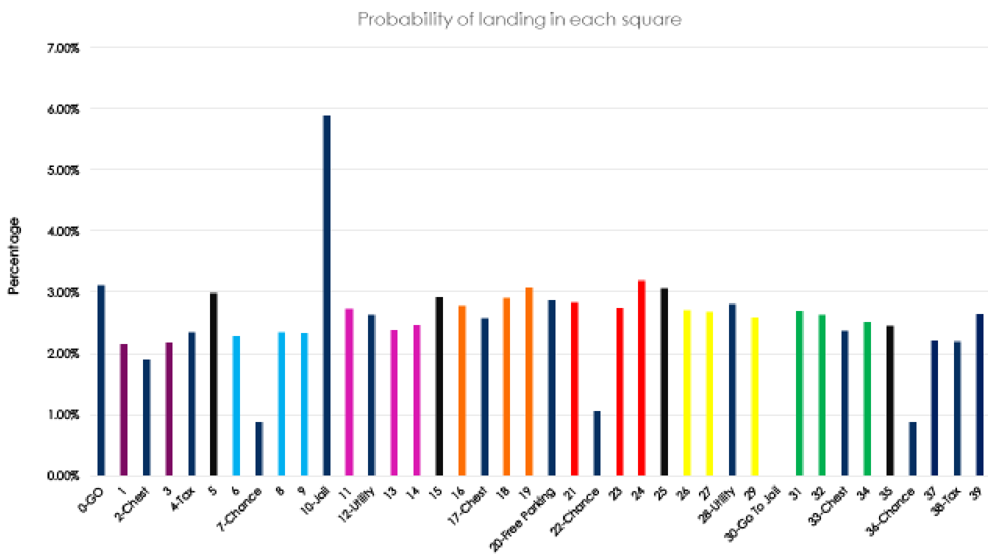


The first thing we need to realise is that since we are using two dice to move around, not all outcomes have an equal probability of being rolled.
For example, there is a greater chance you will roll a seven than any other number. This is because there are more combinations that give out a seven than anything else.

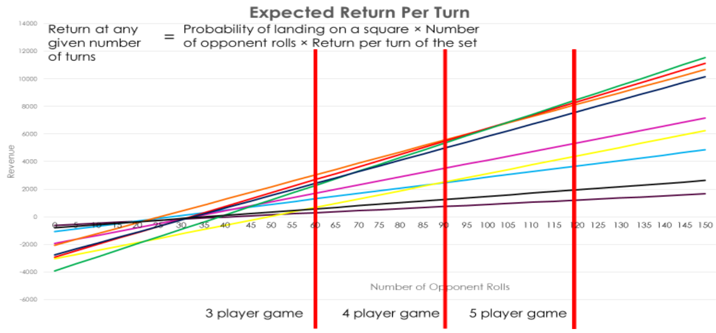
Dice Roll	Combinations	Probability
2	(1.1)	2.78%
3	(1.2),(2.1)	5.56%
4	(1.3),(3.1),(2.2)	8.33%
5	(1.4),(4.1),(2.3),(3.2)	11.11%
6	(1.5),(5.1),(2.4),(4.2),(3.3)	13.89%
7	(1.6),(6.1),(2.5),(5.2),(3.4),(4.3)	16.67%
8	(2.6),(6.2),(3.5),(5.3),(4.4)	13.89%
9	(3.6),(6.3),(4.5),(5.4)	11.11%
10	(4.6),(6.4),(5.5)	8.33%
11	(5.6),(6.5)	5.56%
12	(6.6)	2.78%

So, it would be a fair assumption to say that after your first roll you would end around squares 6-8 and most probably on square 7.
However, that is not all. “Monopoly” has some twists. For, example, the Community Chests and Chance Cards, which may reward you with money, make you pay a fee or sent you to another square. There are 16 Community Chest cards out of which, one sends you to GO and the other one to JAIL and 16 Chance Cards which may send you to GO, to JAIL, to squares 5,11, 24, 39, 3 squares back (4, 19, 23), to the nearest station (5,15,25,35) and to the nearest utility (12,28).
Now, taking all of that into consideration we will begin to calculate the probability of landing on each square. Imagine a pawn, starting on GO. On the first roll, it is split into small pieces and distributed around the board based on the probability of ending up at each square after the end of the first roll.
1.22% would still be on GO, after being sent back there from a Community Chest or a Chance card and 6.60% would be on jail, either just for a visit or behind bars. We continue the same process, splitting the pawn into small pieces and distributing it around until the part of the pawn that comes into the square is the same as the part leaving that square. This means that we have found the probability of landing on every property.

Here are the results:



As you can see the most visited square is JAIL, followed by square 24, GO and then square 19. So, since you can’t buy JAIL or GO the best properties should be square 24 and 19… Well, you would be partially correct to assume that.
However, we need to investigate what kind of return should you expect from your investments. We will look at the profit for each property with a hotel on it and since a set is required to build that hotel, we will investigate the set as a whole. We can find the average return per roll of a property by multiplying the rent of that property by the probability of landing there.
As you can see the best set changes depending on the number of rolls.



An average game takes 30 rolls per player therefore the oranges would be the best option for a 2-3 player game, but the reds become better for a 4-player game and for 5 players or more the greens are your best bet.

CONCLUSION

To recap:
For Risk, Australia is the best continent to control in order to boost your army in the early game. When you know battle is inevitable, attack as soon as you have the same number of attacking troops as your opponent has defending troops, you hold the advantage. And the larger the battle the larger the attacker advantage. For Guess Who, the broad questions are the best. Always ask the broadest question possible and since you have the better strategy try to play multiple rounds to ensure that you come up on top. Finally, for Monopoly, for a 2-3 player game go for the oranges, for a 4-player game go for the red and for a 5+ player game go for the greens.

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DIDO AND HER PROBLEMS

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ABSTRACT

Dido's problem of maximizing the area which is enclosed within a certain perimeter is probably the oldest problem of optimization. It is also known as the "Isoperimetric Problem". In antiquity, Zenodorous gave a geometrical solution to this problem using polygons. Zenodorous is conciderd by Constantin Carathéodory to be the founder of Calculus of Variations. Twenty centuries later Steiner gave his own geometrical proof. Other mathematicians, such as Weierstrass, solved the Isoperimetric Problem using the tools of Calculus of Variations. Nowadays Dido' s problem is an active field of research in several areas of Mathematics.

INTRODUCTION

If we are to learn something from the history of mathematics is that very often, the most unique and important mathematical ideas and discoveries emerge from simple problems. These problems could be referring to everyday situations such as the need to construct something with certain specifications. A characteristic example is that of the isoperimetric problem, a classical problem of shape optimization with great impact on society and the production of new mathematics.

The word "iso-perimetric" is consisted from the Greek word "iso" which means equal and "perimetric" which refers to the perimeter, the length of the boundary of a two-dimensional figure. Consequently, the isoperimetric problem is formulated as following:

"*Find a closed plane curve of a given perimeter which encloses the greatest area*".

THE MYTH

"The Kingdom you see is Carthage, the Tyrians, the town of Agenor;
But the country around is Libya, no folk to meet in war.
Dido, who left the city of Tyre to escape her brother,
Rules here--a long and labyrinthine tale of wrong
Is hers, but I will touch on its salient points in order...Dido, in great disquiet, organised her friends for escape.
They met together, all those who harshly hated the tyrant
Or keenly feared him: they seized some ships which chanced to be ready...
They came to this spot, where to-day you can behold the mighty
Battlements and the rising citadel of New Carthage,
And purchased a site, which was named 'Bull's Hide' after the bargain
By which they should get as much land as they could enclose with a bull's hide."

Virgil in “Aeneid” narrates the myth of Dido, the beautiful princess of Tyros who left her country, as she was trying to escape from her brother, and went to North Africa, in Lybia. After arriving to Carthage, she found herself wanting to build her own city within the country. After talking with the king, she was given the chance to do so as long as the city was as big as the skin of a cow. After cutting the skin into thin strings, forming a rope, she was confronted with the isoperimetric problem. What shape was she supposed to form using the rope, in order for her town to cover the largest area possible?

ISOPERIMETRIC PROBLEM IN ANTIQUITY

According to Blasjo (2008): “The Greeks pretty much solved isoperimetric problem, by their standards, when Zenodorus (200 BC - 140 BC) proved that a circle has greater area than any polygon with the same perimeter. His work was lost. We know of it mainly through Pappus and Theon of Alexandria”. Zenodorous is a Greek mathematician who probably lived during the same era as Archimedes. His work was relevant with isoperimetric figures. Constantin Carathéodory considers Zenodorous to be the founder of Calculus of Variations. This statement indicates the importance of the first steps being made in a field of mathematical research even though the “pioneer” might not have the slightest idea of where that could lead.
Subsequently Blasio presents Zenodorus polygon proof:

Theorem. *For regular polygons with the same perimeter, more sides imply greater area.*

Proof. Consider the apothem, the radius-like perpendicular drawn from the center to a side (see Figure 1).

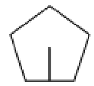

Figure 1.

Half the product of the apothem by the fixed perimeter yields the area of the polygon:

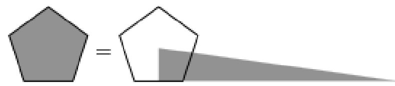

Figure 2.

The apothem is the height of the triangle in Figure 3:

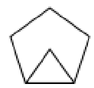

Figure 3.

If we increase the number of sides, the base of the triangle in Figure 3 is shortened and the angle is decreased. It is clear that its height increases. We would prove this by trigonometry; Zenodorus had to rely on the usual pretrig bag of tricks. It is routine for us, and it probably was for Zenodorus as well. ■

The idea of this proof is very simple: Since the perimeter is fixed, the apothem determines the enclosed area. So the area increases when the apothem becomes bigger. This happens when the number of sides is increased.

Yet, it seems that ancient Greeks were aware of the solution of the isoperimetric problem even earlier. John Philoponus (490 AC- 570 AC) says that Plato supported that circle encloses the maximal domain among all two-dimensional shapes with the same perimeter. Philoponous also comments Aristotle's belief that among isoperimetric shapes, greater area is enclosed by the one that has the greater number of angles.

THE ISOPERIMETRIC PROBLEM IN THE NINETEENTH CENTURY

In 1838 Jacob Steiner gave his own geometrical proof for the isoperimetric problem (Blasio, 2005):

Steiner's four-hinge proofs.

Theorem. *Any figure with maximal area must be a circle.*

Proof. Take a figure with maximal area. Cut its perimeter in half with a line. This line will split the area in half as well, because if it did not we could take the half with the greater area together with its reflection in the line and get a figure with the same perimeter but greater area (Figure 11).

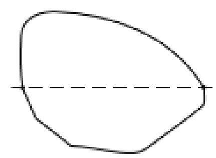

Figure 11.

Consider one of these halves. Suppose it is not a semicircle. Then there will be some point on the boundary where lines drawn from the points on the symmetry line meet at an angle that is not a right angle.

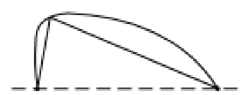

Figure 12.

Think of there being a void inside the triangle and think of the pieces on the sides as glued on. Slide the endpoints along the symmetry line to make the angle a right angle (Figure 13).

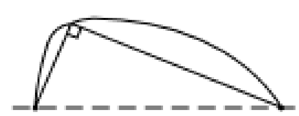

Figure 13.

This increases the area, so reflecting this gives a figure with greater area while the perimeter is still the same. That is impossible, so the halves must be semicircles and our figure must have been a circle to begin with. ■

In fact, Steiner gave five proofs. The problem is that in all proofs he assumed the existence of a solution (his strategy was always to take a figure that is not a circle and show that its area can be improved). This led to many contemporary mathematicians criticizing his proofs:
Peter Dirichlet: "Jakob, you have an incomplete proof. You have assumed that a solution exists."
Jakob Steiner: "Peter, I have a valid proof and I'm not going to let you rain on my parade, please go away."

Perron, (1913):

> **Theorem.** *Among all curves of a given length, the circle encloses the greatest area.*
>
> *Proof.* For any curve that is not a circle, there is a method (given by Steiner) by which one finds a curve that encloses greater area. Therefore the circle has the greatest area. ■

> **Theorem.** *Among all positive integers, the integer 1 is the largest.*
>
> *Proof.* For any integer that is not 1, there is a method (to take the square) by which one finds a larger positive integer. Therefore 1 is the largest integer. ■

Proof of isoperimetric problem using the tools of calculus of variations

According to Blasio (2005) many mathematicians, such as Weierstrass, gave their own proof. The basic idea is this *"Take a curve and wiggle it a little bit, while keep in gits perimeter fixed. If the curve is the one with maximal area then we are at an optimum, so an infinitesimal wiggle will cause zero change in the area. In order to find the optimal figure, therefore, we calculate the change in area caused by an infinite simal wiggle and set this equal to zero. This leads to a differential equation that must be satisfied by an optimal figure, and indeed that is satisfied only by circles".*

IMPACT ON CONTEMPORARY MATHEMATICS

Isoperimetric problem is an optimization problem. Professor Taria mentions that *"Optimization problems are relatively easy to understand when compared with problems in many other branches of mathematics. Controversy invariably leads to interest. Hence, important optimization problems embedded in some controversy have played major roles in motivating and promoting mathematical activity".* The fact that we are discussing about an active field of research in several areas such as differential geometry, discrete and convex geometry, probability and Banach spaces theory shows the importance of this early posed problem.

CONCLUSIONS

The isoperimetric problem, the first ever problem to be created in the field of maximization, wasn't a source of trouble for just Dido. To the contrary, it became a topic of study for many important mathematicians who either solved it or at least tried to. Subsequently, it is concluded that mathematical breakthroughs are not necessarily a result of complex problems. It is possible that simple, everyday hurdles become a source of inspiration for research and great mathematical discoveries.

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STUDENT PRESENTATIONS IN SCIENCE

CARTOONS IN PHYSICSLAND

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ABSTRACT

“What would cartoons be like if physics laws were in effect?”
The truth is that all of us have spent hours on watching animated cartoons. We have all laughed with the heroes’ blunders. We have all felt anguish while watching Sylvester chasing Tweety orTom chasing Jerry and we have all felt joy while watching Elmer razed to the ground in his effort to catch Bugs Bunny. Innumerable times, we have seen the heroes experiencing complete disaster but in the end, they always came back to their former shape, safe and sound.
No wound, no scar or movements and motion in general, were affected by gravity. We could watch the heroes’ thoughts suddenly take shape and acquire entity skills. They could also become completely flat. It seems that for the animation cartoonists no physics law exists.
But what would happen if they followed those laws?

How would animation be if the laws of physics were in place?

The truth is that we have all spent hours watching cartoons and we have all laughed with the heroes' goofiness. We all feel angry when watching Sylvester chase Tweety, or Tom hunting Jerry and of course we all feel joy watching Elmer Fan falling on the ground while trying to catch Bugs Bunny. We have seen the heroes, countless times, to experience complete destruction, but in the end, they have always returned to their original form, safe and sound. No trauma, no scar.

Their movements are not affected by gravity. Their thoughts suddenly getting into existence and became matter. They can become completely flat, disappear and appear from nowhere. It seems that cartoons defy the laws of physics.

But what would happen if they followed these laws?

The laws of physics govern our lives and no one can defy them except animation.
Cartoon physics is a playful system of rules that replace the usual laws used in cartoons to achieve a funny and hilarious effect. In particular, the laws of physics are objective and sturdy whereas the corresponding laws of animation are subjective and volatile.

Many of the most famous American cartoon film companies, especially Warner Bros. Studios and Metro-Goldwyn-Mayer, unconsciously developed a set of such "laws" that have become the foundation of animation. Art Babbitt who worked at Walt Disney Studios as an animator said, "Animation follows the laws of physics- unless it's funnier otherwise."

A set of such rules was first recorded by Mark O'Donnell in 1980 and then enriched by others as they passed through the years. This catalog was originally known as "Cartoon Motion O'Donnell Laws", published in Esquire in June 1980. In 1994, the new enriched version of the Institute of Electrical and Electronic Engineering was published in his magazine and is why it has helped to spread as well as to perfect it.

I will try to present the laws that apply to cartoon with examples, mentioning which laws of physics do not apply, but also some movie parts where Mark O'Donnell's laws are not valid. As is well known, there is an exception in every rule

1st Law of Cartons, and the most important

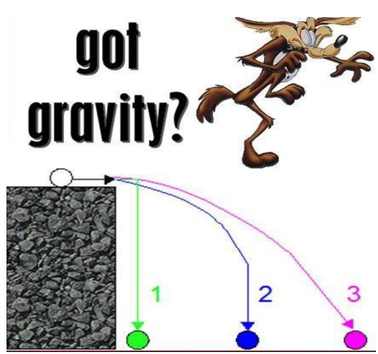
A body will remain in place or in motion in the air until it becomes aware of its condition. Once it realizes it is in the air, gravity takes action and falls to the ground. This is known as: "Defying the law of gravity"

All of us have seen the scene that Road Runner fools the Coyote and the last always ends up on the ground after he has a minute to realize that he’s on the air.

What would happen if an object was to fall from a cliff according to physics if we ignore the air resistance that is virtually 0?

1. We have the following 3 options. And I imagine a few from here could find the right one.

2. The object will stop in the middle of the air and then drop vertically downwards.
   The object will fall by making a circular track in the beggining and after a point it will fall straight down.
3. The object will continue to move throughout its movement.



The first move is the one that always appears in the cartoons, but the correct answer is the third one.

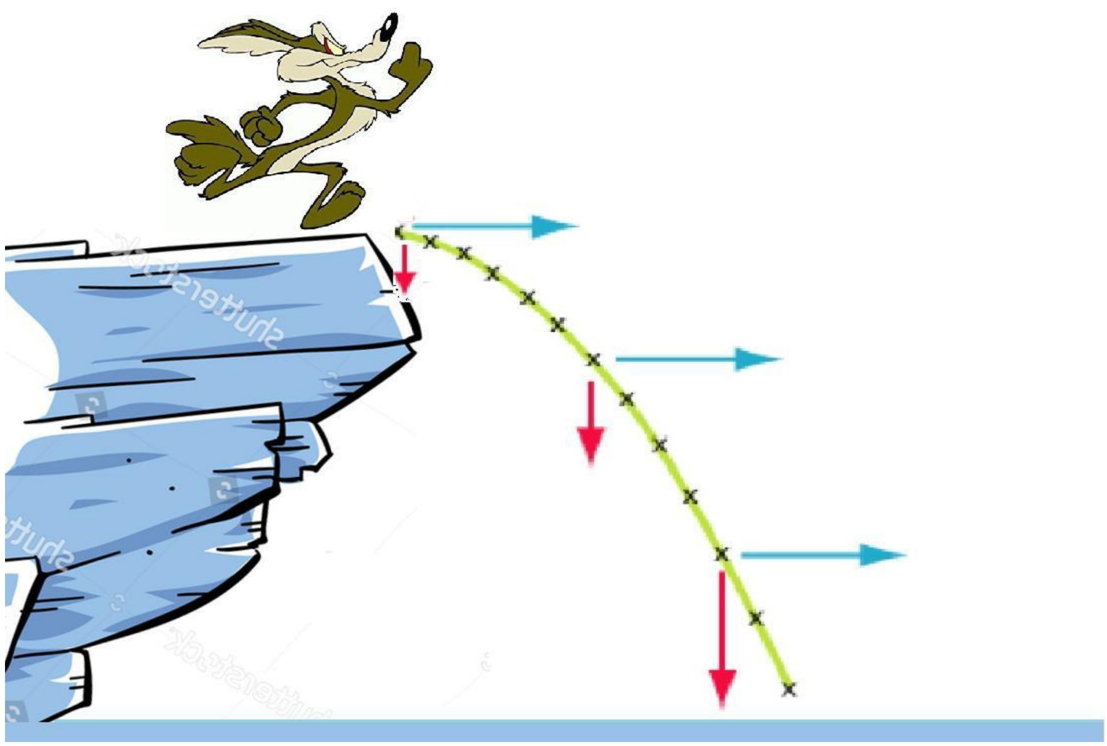
What laws apply to the object's movement?

All the forces acting on it are balanced. So we are in equilibrium, meaning the sum of all forces on it is 0 ($\Sigma F=0$)

- According to Newton's first law or otherwise the law of motion (or the law of inertia) an object at rest stays in a resting state until a force acts on it. An object in motion continues to move (linear motion and steady speed) until a force changes this movement

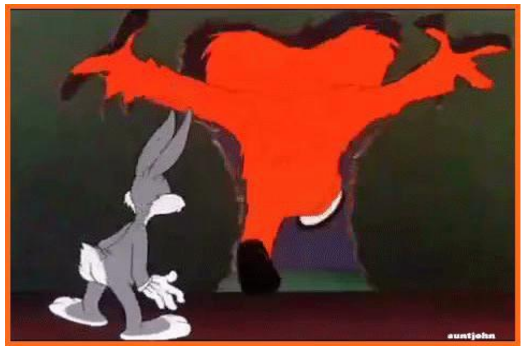
In this example, the force acting on the Coyote is gravity, which is exerted on every body regardless of whether it falls, is raised or moved. The weight has a vertical direction.

The blue arrows on the horizontal axis in the above image depict the speed of the Coyote uo and the pink darts on the vertical axis the gravitational force. So the path to be followed is designed with green, and that is the result of the above forces. So we can say that we have a horizontal shot, meaning a complex motion consisting of 2 simple ones.



Silhouette of Passage

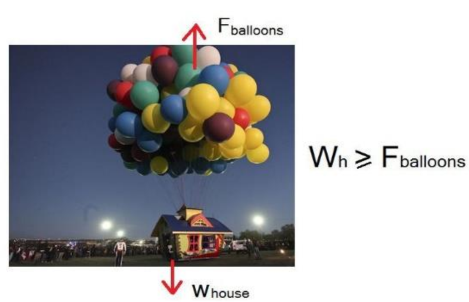
Everybody in the cartoon, while running, can penetrate with its body any object or physical obstacle (wall / snow) leaving the print of its silhouette on it and this in the cartoon world is known as the "silhouette of passage". But in the world of physics this would not be possible. Let's see what exactly plays a role and how it will become reality.



In this example Gossamer (the orange monster) and the bullet have a mass “m” and runs at a velocity “u”. In general, the behavior of a moving body depends both on the mass “m” and the velocity of “u”. So, to study this case of “Collision” and others similar to this, we introduce another vector quantity called momentum P and expresses the product of the mass “m” of the object and its velocity “u” and is given by the formula: P = m * u. So we can study collision phenomenons like this and the following example in which we shoot an apple with a gun and the bullet passes through it. But what is their difference and the first is almost impossible while the second one actually is? The speed… The velocity of the ball is far greater than that of Gossamer, so it would realistically be impossible for him to penetrate the snow or the wall.

Balloon House

Most of us have seen the Up movie and we remember the wonderful scene that the house takes off using balloons. But can this eventually become a reality?
Helium balloons have a lifting force of 1 gram per liter. A normal balloon may have a diameter of about 30 cm. The formula to calculate how many liters of helium can fit into a sphere is: 4/3*π*r3 where r is its radius. Thus, a normal balloon can lift about 14 grams, assuming that the weight of the balloon and the string itself is negligible.



A body weighing 50 kilos, which means 50,000 grams, will need 3,571.42 balloons to barely get of the ground. In order to make sure that this mass will fly, we will need 500 balloons more, so: 4000 balloons total.

Lets take look from the perspective of physics on that:
Newton's first law applies if the balloons are exactly 3.571.42 . That means that total force is 0, $\Sigma F = 0$. If there are 4000 ballons then we have Buonancy. Buoyant force =object's weight - apparent immersed weight. That way the mass can fly. Disney managed to lift a home using 300 huge balloons.

All these examples are creation of our own imagination but as we can see there is hope that maybe someday with the help of science, the cartoons will become reality. What would happen if could run with the speed of the Road Runner or walk in the air? Time will tell!

EVOLUTION OF SPECIES AND ENTROPY

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ABSTRACT

It is a common misconception that the evolution of life contradicts the 2nd Thermodynamic Law. We argue that such a configured phenomenon as species evolution could not but be in accordance with the laws of physics.
The 2nd Thermodynamic Law states that in a closed system entropy always increases. For the purposes of our study, we consider entropy as a measure of energy disorder and we will take into account that thermal energy is the most degraded, disordered, hence highly entropic form of energy.
The Earth is an open system which constantly receives solar energy. During this process, while life evolves, living organisms may store energy in an orderly, low entropic way, but at the same time the sun entropy increases, while living organisms and functional ecosystems release highly entropic thermal energy. We may say that in a non-equilibrium system such as Earth, the variable species are the dissipating structures, with the ultimate goal of solar energy degradation to a disordered thermal form.
Considering the aforementioned, life is not only compatible with the 2nd Law of Thermodynamics, but also necessary for its application.
Despite the various approaches to determine whether evolution has connection with physics, from Boltzmann to Schrödinger and Eiger, there is still one question that arises. Since Earth and the evolving terrestrial life is a non-equilibrium system can entropy really be defined? Or maybe there should be some other parameters to estimate, in order to clarify this long-term debate about life being an improbable occurrence or an inevitable path given the circumstances of its origins.

Theoretical Approach

A. Introduction

Living organisms are highly organised and orderly structures, derived from a primodial soup of elements and simple organic molecules and evolved following two main paths as described by the Darwinian theory of Evolution to gradually more and more complex structures, such as human, the most complex organism known today.

All these are a nicely put theory, not really difficult to understand and in general a quite easy topic to wrap our heads around. But what happens when physics enter the topic? Does physics clarify the origins of life and evolution?

The 2nd Law of thermodynamics suggest that, spontaneous processes always cause an increase in entropy, or simply tends to increase disorder.

Well, living organisms evolved to such orderly structures from a chaotic pool of elements and simple molecules, while physics say that this is impossible. Does this imply that our whole existence is impossible?

The scientific community is still in a debate considering the topic. For the purpose of our study we tried to gather some of the current scientific aspects on the subject and lean to the conclusion that life does not violate the 2nd Law of thermodynamics. Evolution may be a seemingly contradicting process, but this does not make life impossible. Improbable maybe, but not impossible. Some scientist also claim that life may be a necessity for the application of the 2nd Law of Thermodynamics imperative.

Before coming to present the discussion of the topic evolution versus physics, we think it is necessary to summarise the basics principles and theories of each scientific domain, since this would give a more complete aspect of the topic.

B. Origins of life

Early earth formed 4 billion years ago and contained the building ingredients of life such as liquid water, oxygen, hydrogen, carbon, nitrogen, sulfur and phosphorus, which are considered the most important. Shortly after the earths formation, life occurred at about 3.8 billion years ago. The main energy source for the necessary reactions and transformations was of course the solar energy. The most popular and widely accepted theory for life formation is Oparin's theory of the primordial or primeval soup, according to which the first simple organic molecules formed and were later transformed or reacted to synthesize more complex organic molecules, such as proteins, carbohydrates and of course nucleic acids, in an oxygen-less atmosphere, using the solar energy. Here we are talking about order and organised structures again.

C. Evolution of species

Darwin describes the evolution of species as a process called natural selection and his observations are driven to 3 basic conclusions.

- A survival battle occurs among the organisms of a population

- The outcome of the battle is not circumstancial, but depends on the characteristics that each organism has heredited. Organisms with traits suitable for the current environmental conditions, are those with the higher survival rate, i.e "survival of the fittest" as it prevailed to be said.

-The suitable or helpful traits, we may say, seem to be heredited more frequently, but of course this is a result of the survival of the fittest. The organism that survives, reproduces and the "good genes" go on to the next generation.

It is noteworthy that the unit of evolution is a population and not a single organism.

Evolutionary theory affects populations, not single members of a population. A member may appear one new trait at most but evolution in order to happen needs accumulation of new hereditable traits.

Going on from the yet "orderly talk" about living organisms, we are talking about physics and embracing chaos.

D. Entropy and 2nd Thermodynamic Law

We just skipped in a far more complex and intriguing topic.

We will try to explain what is entropy, or more specifically, what we consider entropy for the purpose of our study, since this term alone is a conflicting one for the scientific world.

Entropy is usually defined as a measure of disorder, but that is a vague definition, that it is misconcepted by the vast majority of students, but also scientists.

We try to clarify entropy definition, as used in our study, through simple examples.

Let's imagine a meadow with 3 sheep grazing happily. Let' s also imagine that this meadow is divided in three land plots. The sheep move freely from one plot to another. If we take some time to figure out all the possible configurations for the sheep, we will note that there are 10 possible configurations, as seen in figure 1.

A closer look to the possible combinations reveals, that there are 7 combinations where the sheep are spread to more than one land plot, while there are only 3 "orderly" combinations where the sheep are located only in one plot.

We automatically conclude, that it is more possible to see the sheep spread out at a certain moment, than to see them all together grazing in one plot.

This is exactly what happens with energy. Energy tends to spread out. There is no specific driving force for this process. It is just that when, sheep, thing, units are spread, there are more possible configurations for them to be, this is what we call "microstate". In the case of the sheep, there are 7 microstates where the sheep are spread out, but they all correspond to the same "macrostate", we just see sheep spread out. This spread out image is a disordered one. It is a state that is highly entropic. In others words, according to the aforementioned, it is a state with more configurations and therefore a more probable outcome.

So, returning to energy, when energy is spread out we are talking about a high entropy state. An why energy spreads out? Simply because there are more possible ways for the energy packets, called quanta, to be placed among the particles of the matter.

The 2nd Thermodynamic Law requires an entropy increase during a spontaneous process. A spontaneous process is a process that does not require work to be done in order to occur.
Does entropy always have to increase?

Well, an increased entropy state, as we mentioned is a very probable one. As the number of the sheep and the numbers of the plots increase, so do the possible configurations for the sheep. Same goes for energy and particles. As the energy packets and the particles increase, more and more possible way for the parcels to be placed to the particles arise. Since the whole number of possible configurations increase, the chance of the "spread out" configurations also increases against the chance of the "orderly" configuration. But it is useful to remember that a chance always remains a chance, this means that there is always a possibility for an unlikely events or process to occur.

Extending our examples, let's see something closer to our used definition of entropy.

Imagine a glass of liquid water and a glass of ice. If one was asked about which glass he thinks is more entropic, the answer might be the glass with the ice, because our vision is a tricky sense and may influence our reasoning. In order to be able to make any correlation between entropy and evolution, we always need to think in terms of energy configurations. The liquid water has more energy, since the liquid molecules are more mobile, so there are more energy configurations in the liquid state of the water, i.e. there are more ways for energy to spread out. According to that, one may say that entropy is a measure of energy spread out. Heat or thermal energy is the most entropic energy form since it is characterised by intense movement of matter particles and has a great tend to spread, hence heat moves from a hotter to a cooler object. But since it is all about energy configurations, how come heat flow only from a hot object to a cold one? Theoretically heat can also flow from a cold object to a hot one, but this just does not happen because of the size of the system. As the size of a system increases, meaning atoms, molecules and energy packets increase, so does the probability of the "spread out" configuration, against the probability of the "orderly" configuration, which becomes extremely improbable. Such a path of thought helps us woven out the usefulness of entropy. Entropy can explain why some processes only go one way. Heat flows from hot to cold objects, air leaks out of a punctured tyre, sheep are distributed all over the grazing area and we only evolve, we haven't seen a species un-evolve, but can entropy really apply to living organisms? We made an attempt to ascertain whether such a relationship exist and if so, we tried to figure out if these to concepts, entropy and evolution have a contrasting relationship.

E. Energy degradation

A term usually met in this kind of arguments is energy degradation. Heat is the more degraded form of energy, which means that it is difficult to convert thermal energy to another more useful energy form. Heat usually appears as energy loss from a system. While driving out cars for example, chemical energy of the fuels is converted to motion and that is the useful work we need to be done, while heat generated in the car's engine is just energy loss, therefore we call thermal energy degraded and when we talk about energy degradation, in essence, we are talking about various kinds of energy transformed to heat.

Let's think for a moment, why is heat degraded and why it is difficult to convert it to other energy forms. But because energy is as spread out as energy can be, or an energy form with great spread out tendencies, hence highly entropic, so more probable state.

Discussion

Having explained all the necessary terms and concepts we can move on and examine the debated matter of an existing contradiction between the 2nd Law of Thermodynamics and Darwinian evolution.

Some inquisitive mind have wondered, and still do, about what is life. Schrodinger in 1943 issued a book, "What is life?", trying to shed some light on this topic.

Schrodinger states that although life evolution seemingly defies the 2nd Law imperative, in reality one does not exclude another. After all, the 2nd Law applies in isolated systems, while living organisms exist in a world of constant energy ang matter fluxes, meaning a non-equilibrium system. Entropy is actually defined in a closed system. This statement indicates the need for a different, maybe more "elastic" approach of the law. Schrodinger' comments were food for thought for his subsequents.

The more simplistic approach which suggest that life does not violate the 2nd Thermodynamic Law, emphasizes on the fact that Earth is an open system, which constantly receives solar radiation. Fusion processes in the sun's core release gamma-rays as a by-product. Gamma-rays are transformed to the solar energy finally emitted towards earth, by the time they reach the photosphere of the sun, which consists of visible light, infrared radiation and ultraviolet radiation.

This theory further suggest that despite the decrease of entropy on earth due to evolution of species, the 2nd Law is not violated because the total entropy of the sun-earth system increases. In other words the sun's spreads its "orderly" packets of energy, increasing its entropy, while the outer space absorbs solar energy and is heated therefore also increasing its entropy. Not to mention that life on earth, despite its organised form, it also produces heat, also causing an entropy increase. These processes are thought to cause such a big increase in entropy, that compensate the small decrease cause by living organisms and their evolution. This opinion is also supported by Styer (2008) and by Bunn (2009). Both scientist made some hypothetical calculations of the entropy decrease due to species evolution and concluded that this decrease is so small, that is compensated by the overall entropy increase in entorpy of the sun.

Sewel (2013) on the other hand, thinks that the compensation argument is invalid and does not refute the fact that evolution is still an improbable occurrence.

Styer' and Bunn's approaches, also supported by many other scientist, all make the same assumption, that all that is needed, so that the 2nd Law still applies, despite evolution, is just an external energy source, the sun. Sewel states that the external energy source and its entropy increase, still does not explain how or why the decrease of entropy because of evolution occurs. He also states that the 2nd law is all about predicting the most probable macroscopic outcome, while taking into account what happens at a microscopic level. (Remember, the most possible outcome is to observe heat transfer from a hot object to a cold object, because there are more energy configurations this way and this makes the heat flux from hot to cold more probable). He also uses an extreme paradigm of a tornado hitting a town and turning it into rubble. Then a second tornado hits the same town and turns the town back to its initial form! The case of the second tornado is a violation of the 2nd law that defies common sense. But still talking about probabilities, anything can happen, even if we are talking about something that none of us will see in a lifetime. This way Sewell wants to underline that the simplistic approaches, as described above, cannot apply to living organisms in a way that could really describe the phenomenon of evolution and the origins of life.

A more integrated approach was given by Schneider and Kay, in 1994, although it was not given the appropriate value and has not really been discussed.

The authors of this paper relied on a restatement of the 2nd law in order to explain how the beginning of life is really in accordance with physics.

The reformulated 2nd law says that when a system tends to resist or even "neutralise" the gradients which are submitted on them. A simpler way to put this is that, under certain circumstances, organised structures can emerge, in order to help the progress of a physical process.

An everyday example of these emerging organised structures is seen when we are washing a bottle with a narrow orifice. We fill the bottle with water, turn in upside down, so that the soap is washed off. Leaving aside difficult terms such as entropies, energies, systems etc for a moment and just focusing on the expected, on the physical we may say result, what do we get? Just water flowing down from the bottle to the sink. Let' see what will happen, if we are in a hurry, while washing this bottle and give it a little twist. A mini vortex will appear and the bottle will empty much more quickly. What happened there? We gave a little energy and water molecules organised in a highly ordered structure of numerous molecules combined, just to help the water flow to a lower part and all of these just seem so logical, so "natural".

Using this simple observation as an analogy, we can now think of living organisms as dissipating structures! Energy flows from the sun towards earth. Energy also tends to spread and the energy form with the greatest tend to spread is heat. That may be considered the "natural" process, as mentioned above, according to the laws of physics, not to forget entropy increases, disorder increases. How? Through the spread of heat. Now it starts to make sense. Life is for the sun-earth system the same thing as the mini swirl observed in the bottle. Living organisms help the flow of the incoming solar energy and its degradation to heat, the most disordered and highly entropic energy form. In other words, evolution helps the application of the 2nd Law of Thermodynamics.

The initial statement of the law is quite restrictive, since it applies to closed systems in an equilibrium state, without energy or matter fluxes. In context of the restated law, life is a dissipating structure, formed within a non-equilibrium system, to help the solar energy flow and its degradation to heat. The ultimate "goal" of a system, is to reach at an equilibrium. And what can be considered equilibrium for our terrestial circumstances? The equalization of temperature, according to Boltzmann, achieved by the heat spread out. And why is that? Because a spread state for the energy is the most probable one, as explained above. As simple as that, a matter of chance.

Summing up, order can arise from disorder in a simple physical systems, if this is for the purpose of helping the system reach an equilibrium, do the expected natural thing. Highly organised structures can emerge in order to neutralise an incoming gradient. This way from an element soup the first organic molecules were formed, which gradually transformed into more and more complex structures, according to Oparin's theory and under the continuous influence of solar energy the first living organisms arose. From then on, evolution of these organisms took place in order to create more solar energy dissipative pathways, resulting in its degradation, its spread out. From this aspect, evolution seems to be an inevitable phenomenon, necessary for the application of the 2nd Thermodynamic Law imperative, as Eigen has stated.

The well-known darwinian "survival of the fittest" driven by environmentally appropriate hereditable traits and maybe some random "good" mutations can now be seen as survival of the species which are more effective into dissipating energy and thus producing entropy. The more developed the living structures, the more energy they will be able to degrade.

Biological systems develop in a manner as to increase their energy degradation rate. Evolution occures for the shake of new dissipating pathways formation. Species diversity and even ecosystem growth can now be attributed to solar energy, as the more simplistic approaches describe, but now it makes more sense and might be an answer to Sewell's justifiable speculations about life being an improbable occurrence.

Ecosystem development increase energy degradation, thus follows the 2nd law of thermodynamics. More biomass means there are more dissipative pathways, while species diversity can be translated in means of more different pathways, all for one purpose, energy spread out and entropy increase.

From a clearly biological point of view, life is more abundant where food and energy sources are profuse. Life thrives at the equator tropical zone, but isn't that the region where the incoming solar energy is the greatest? More sun, more dissipating structures needed, more life. Under this path of thoughts, the conjugation of biology and physics is achieved. Life and evolution, not only does not contradict the laws of physics, but emerged because of them.

Conclusions

We still aren't able to verify a relationship between evolution and entropy, but given a try to really understand the entropy, we think that these two concepts are not really that incompatible. From oversimplified approaches and calculations by Styer and Bunn, to reasonable questioning of life's improbability by Sewell, the idea of dissipating structures described by Schneider and Kay looks like a quite solid and complete description of the reasons for life's origins and evolution. The most important achievement of this theory is the successful conjugation of the two sciences.

So is life an improbable phenomenon, an impossible occurrence, in complete accordance with physics or even a necessary incident? That is a question that still, no one can answer with certainty. We what we actually noted through our study, is that, when keeping an open mind about specific scientific terms, such as entropy, then the two sciences do not seem so incompatible.

Schrodinger was one of the first to put the question of what life really is, in a scientific way. Our guess is that we are still far away from such answers.

At this point we would like to quote some words of an enlightened modern day physisist, Aatish Bhatia, "The story of our universe is that of climbing Mount Entropy. Both the base and the peak of the mountain are inhospitable to life, but somewhere along those rising slopes, the conditions were just right for complex and wondrous things to emerge, things like trees and jellyfish and heartache and cheesecakes "

Maybe we are just a swirl in a bottle charged with an Aristotelian "final cause", with a duty, to drive the universe to energy degradation, to an equilibrium. Since that sounds a little sad, we prefer Bhatia's approach of life as a wondrous journey.

From there on we are entering the real of philosophy, and philosophy is clearly a matter of perception.

Acknowledgments

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Video of Jeff Phillips, giving every day examples of entropy were also of great help for the completion of this study.

FUTURISTIC HUMAN VISION WITH LENSES

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ABSTRACT

Glasses have already 'invaded' in everyday lives due to the continuous use of technological devices and gadgets, which cause a lot of eye strain. Furthermore, many people are taking advantage of their aesthetic presence and incorporating them into their entire look. However, this eye strain, ends up developing refractive errors on the individuals' eyes. Contact lenses, prescription frames, and refractive surgeries are all together aiming to correct all the abnormalities that are caused from the refractive errors (myopia, hypermetropia, astigmatism, presbyopia, a.o.), thus providing better vision quality.
- Do they address the problem at its roots?
In the future, there is a possibility that our glasses will be adjusting themselves for different purposes of use, for example we will able to focus on an object several kilometers far away, see through solid structures like X-rays, they will protect our visual system from radiations either natural e.g. solar or man-made e.g. nuclear power, e.t.c. Our contacts will send an SMS when our eyes need hydration or the eye pressure is above the set limits.
- How will optics, ophthalmology and human eye system biology develop in the following years?
This presentation will be addressing general informative content about lenses, and refractive errors, and we will all disodiscuss the future of human vision.

FUTURE ASPECTS ON GLASSES AND HUMAN VISION

INDRODUCTION

Optics is the branch of optics that involves the general behavior and the properties of light. Light provides us the opportunity of the visual understanding of the world we live in. Without our eyes we could see nothing but an empty black, being unable to admire nature's miracles, the life itself. Today we will discuss the way we see, the refractive errors of our optical system and some futuristic aspects of human vision.

LENSES

First and foremost, the light is a wave itself and needs to be altered in order to be 'used' properly by almost all beings. This is when a lens takes up a role in the game! A lens has the ability to change light's direction in a specific order. Futhermore, there are two major lenses types convex and concave. A convex lens can converge or concentrate the light onto a certain point and its called positive+, while a concave lens takes the light from a point and diverges or spreads it out to another and its called negative-. These two take their power of concentrating or spreading out from their structure and curvature and they seem to appear spherical. Either they have a wider side in the middle or on the edges. We measure lens power in diopters (equivalent 1m of light direction change per 1D). That curvature gives them refractive power, due to the fuction of

prisms. Prisms are more complicated bodies that again have the ability to alter light's direction and break the beam up to its spectral lights (a beautiful rainbow).
The most common convex lens is the magnifying glass used to enlarge small letters, see something small in detail, etc. Another not really known type of lenses, though they are widely used, are the cylindrical lenses. That ones, concentrate or spread out light in a line and not on a point. They are specifically curved in order to have their power on an axis and not on the whole surface. Additionally, there are some lenses that combine different powers onto their surface, a very useful faction when presbyopia takes place. Now all the refractice errors of one person can be treated with on pair of glasses and not multiple ones. Bifocals combine 2 focal powers, Trifocals combine 3 focal powers and progressive have a more stable change focal powers throughout their surface.

(BRIDGE-CONTACT LENSES)

We talk about lenses as we will use them afterwards in the refractive errors section. As you may have understood lenses are used to cure some visual problems.
Lenses could be fabricated from plastic or glass. Nowadays many people rather prefer contact lenses instead of concentional glasses. Contact lenses do exactly the same job with frame glasses with some benefits like aesthetics, better vision quality for high power problems, wider range of vision not restricted on the glasses surface. Nonetheless, the can be proved to be fatal for some individuals. We will talk about that later.

REFRACTIVE ERRORS AND ABNORMALITIES

Its sure that all people have heard of myopia or nearsightedness. Myopia is the inability to see clearly or in quality over a distance. That happens because when the light enters the eye it focuses in front of the retina, reflecting back on. The eye might be bigger than normal and it follows that light doesn't reach its point on the retina. Otherwise, our natural eye lens is more powerful than needed and thus the light is not properly focused again. There is a specific type of myopia, pseudomyopia and its caused due to poor lighting conditions, long hours of short distance focusing, like computing, typing, etc. Eye lens becomes stiff and less flexible due to all above practices, adapting a myopia finally. During darkness, some people do not see properly, like having myopia, whether they really have it or not. First symptoms of patients start when not being able to see the white board, read advertising panels, driving as they used to and others. The most common practice of myopes is that they to shut their eyes a little in order to see better, successfully. There is no problem with short distances like reading, except in myopias of more that -5D. If myopia is derived from biological causes it starts about 7-8 years old to progress. In many adolescents their myopia starts to thrice and ceases at biological maturity.
Myopia is cured with concave lenses that push light back paroperly onto the retina.In myopias under -1.00 it is suggested that prescription glasses are not used to put eyes to adjust naturally in long distances, though if eye strain and seazuress are caused another treatment takes place. Contanct lenses and surgery precedures can be utilized.
Hypermpertropia or hyperopia or farsightedness is vastly noticed at toddlers and young children, as their eyes are not formed well till that age and the lens is unable to focus light in such a short distance. Hypermetropia is determined by a strenuous constant try to see properly especially in

short distances. Eye strain and headaches follow, as long with vision distortion like missing a line while reading. What is more, light is focused behind the retina providing a bad quality image. Most of the times its goes away without doing anything as the eye becomes larger and more addaptice to all conditions. Convex lenses are used to pull the light forwards on to the retina. Last but not least, untreated conditions of high hypermetropia could even lead to sever damage over the optical nerves and other conditions like amplyopia or neural problems.
Astigmatism exists in every person as our eyes cannot be by nature perfectly spherical, most of the times combined with myopia or hypearmetropia. Astigmatism's cause is not known exactly till now, but the main cause is cornea's (the front part of the eye) shape. Cornea becomes sharper in some parts and less wide in others ending up to a general light misalignment inside the eye its is like saying that the cornea should be round like a football but in astigmatism it's more egg shaped like a rugby ball. Light focuses either in front of the retina or behind, as astigmastism is almost always with the refractive errors of myopia and hypermetropia. Hypermetropia does not occur with myopia as they are contrary cases. Astigmatism regards to a generally distorted sight but the worst problem is on the meridians (axis of astigmatism). Eye strain with headeaches are also induced. It is the most difficult problem to cure as it is in favour of specific cylindrical lenses on the frame or toric conantc lenses curved in position of your eye. High astigmatisms are not operated properly through regular surgery as the part of the cornea is aligned and cut by hand, laser operations are more accurate though a little portion of astigmatism is still left over.
Presbyopia takes place after forties. The muscles that control the lens of the eye become stiff and they do not contract properly for short distances. In most cases patients barely see their newspaper or small letters. They do the characteristical move by getting their whole head far away. Additional focal power is introduced to the patient with a change in prescription in current glasses or new reading glasses. Surgeries are not that effective as an implement lens gets in, though concex lenses work fine focusing the light on the retina.

ASPECTS

Glasses frames come in a variety of shapes, colours and structures in order to compensate each person's need. They provide an aesthetic appearance and they even make a more sophisticated look.
Contact lenses are all disposable and they have an expiration date and certain days of use. They can even change iris colours giving an elegant style to the user. They can be proved to be fatal as an unwise use can lead to severe allergies, eye soreness, infections and finally even death.
What is more and not that researched, is the beneficial use for certain diseases like diabetes and hypoglycemia, allergies, diabetic eye and high eye pressure, conjunctivitis or even anemia!. Nano circuits can be implemented on the silicone lens, working like a small laboratory taking measurements of the eye stability, all the enzymes or percentages of certain substances. For example, tear levels are below the set limits, the lens will connect with your phone sending an SMS alerting you to get some drops instantly. Your insulin injection time can be instructed by the lenses giving you specific details for your sugar lenses.

Some electric lenses like that one in games like call of duty etc, can be focusing on target far away or even zooming on it. The soldier can now see body temperature, water concentration, oxygen supply.
Contact lenses are really effective for the cure of keratoconus also.
Some substances could be fabricated and poured onto the eye surface as UVA UVB radiations become dangerous for our everyday life. Even more, blue light from monitors and smartphone screens, can be reflected or absorbed by that kind of substance.
Furthermore, other lenses could be changing shape constantly in order to ensure the perfect visual acuity for the user.it has been suggested that atropine could prevent the adoption of myopia in teenagers when myopia is firstly recorded about 8-9 years old. Although, the post effects of atropine are not really knonw (expirement Asia).
Glasses that beam X-Rays can provide a wide range of vision within buildings like where something is hidden, helping police forces, or in security forces in airports where guards can wear that glasses and determine some criminals for example.
Karyotype research in molecular level, can prevent the creation of high powers of myopia more than -7D, with targeted chromosomal treatments.

WE ALL NEED TO REMEMBER THAT GLASSES AND ALL KIND OF EQUIEPPMENT USED PROVIDE US A SUBSTITUTE OF REALITY AND THERE IS NO MACHINE THAT CAN OVERPASS HUMAN VISION OR OTHERS' ANIMALS EVEN BETTER THAN OURS. THERE IS NO SUCH THING LIKE PERFECT VISION!

THE MAGIC OF COSMOGENY, A NEVER-ENDING QUEST

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ABSTRACT

The journey began 14 billion years ago… Man's journey though started when the first men - as thinking beings – asked themselves "why?" and "How?". Man has always had the need to explain anything existing, moving or functioning around him. That is the reason why he queried "why" and "how". How was the world created? That is how it all started.
Firstly, theologians tried to explain cosmogeny on the basis of divine or metaphysical forces. On the other hand, common people, throughout the years, turned to myth-making in order to give explanations about the creation of the world. Further to myth-makers, it was the philosophers' turn to give their own interpretations. The first attempt of explaining the world creation philosophically was by Thales of Miletus. Plato and subsequent philosophers as well, based their ideas on the qualities and propensities of the four elements.
In the meantime, the Enlightenment, research and eventually natural sciences bring to light important results and essential conclusions after a series of great discoveries and inventions. Science and technology therefore, after billions of years, make man's long lasting journey reach its peak. The dominating theories on the creation of the world are two. First is the theory of the great explosion and second is the one of superstring theory. Finally, how was the world created? What happens with time, place and any other dimension? Are we talking about a restless circular course of beginning and end, creation and collapse?
In this essay, there will be an attempt for the most of the above questions to be answered.

Man has always wanted to know how he was born and how the world was created. And that is what I am going to present here today. I'm going to present the endless journey of man in order for him to find out how the entire universe was created.

As man has always wanted answers on this matter, he turned to any source available to satisfy this thirst for answers.

That is why we are going to see a series of gradual steps that led him to some explanations on the issue of the world creation.

Starting from Mythology, Philosophy, Theology and Religion and finally Science:

Starting from Mythology, we can refer, for example, to the Chinese belief that the entire universe was concentrated in an egg. All matter was in there with a sleeping giant Pangu. When Pangu woke up, the egg broke and all substance was spread everywhere forming this way the sky and the earth. Later on, the sun was created from Pangu's eye and so on. So, we can easily understand that in the Chinese people's imagination, everything started from an egg, that is to say a central core.

The Egyptians, on the other hand, believed that Atum, a divinity, cried once and her tears, tears of joy, gave birth to mankind. It is easy to see that anything that cannot be explained is credited to divine power.

Now let's go on with the Babylonians. According to their myths, chaos existed everywhere and out of chaos, a God-creator was born. That was Abzu (means freshness) who created with his wife the Gods and then mortal people for hard or manual labor. It is clear here that at that time people needed not only Gods to protect them but also workers to work in the land (for agriculture and cattle-breeding).

At the same time, the ancient Greeks, according to the Orphean myth, believe that the world creation begins from water and from a divinity an egg is born. This egg breaks and the sky and the earth are formed. Here we have the egg again the same as the Chinese myth.

Well, one thing we can only see from all these myths: OUT OF IMAGINATION, THE JOURNEY OF QUEST HAD ALREADY BEGUN AND THE SEED OF DISCOVERIES HAD ALREADY SPRUNG UP.

Not accidentally, Albert Einstein said: **"IMAGINATION IS THE HIGHEST FORM OF RESEARCH"**

And, as the intelligence of man develops, the world creation cannot be explained on mythological basis. Man can realize the inaccuracy and the vagueness of these explanations. It was then that philosophy appeared.

Philosophy was mainly developed in Ancient Greece. Greek philosophers put fundamental questions about the existence of the world and natural phenomena in general.

Democritus, Socrates, Plato and Aristotle are considered to be the pioneers of a more scientific and rationalistic approach to the matter of cosmogeny.

Of course, their ideas had also influence by theological, metaphysical and philosophical extensions. Meanwhile, mathematics is used to help people understand and explain the laws of nature.

After philosophy had put the questions about the world creation and several voices were heard.

There came the time when the first scientists came to the front. For example, some of the most important ones are:

First scientists:

Euclides – the creator of geometry – 3rd century B.C.
Archimedes – famous mathematician, physicist, engineer – about 212 B.C.
Aristarchus - astronomer and mathematician who presented the heliocentric system - about 230 B.C.

It was then that astronomers appeared as well:

Astronomers:

Eudoxus – about 355 B.C. – Plato's student – creator of a geocentric system
Hipparchus – about 125 B.C. – the most famous one that time

Of course, astronomy then could not get rid of the ideological and mythical elements of that period of time.

Nevertheless, the important thing here is to focus on the geocentric model of Plato and the more realistic model of Aristotle, yet geocentric too.

After the 3rd century A.D., comes the Age of Obscurantism, the Middle Ages.
The source of truth is all written in the scripts where nature was created by God and the laws that govern it come from God's will.
This belief automatically pushes aside science and philosophy. It is a time of decline for those two.

Renaissance:

The bridge between the Middle Ages and Modern Times.

Europe makes its first steps out of "darkness" and the birth of a great personality to mark the evolution of science. The Polish monk, Nicolaus Copernicus, develops the idea of the heliocentric system as well as the idea that both the earth and the moon revolve round the sun and other celestial bodies of the solar system, too.

Copernicus was inspired by Plato's and Pythagoras' ideas. In his book "De Revolutionibus", he puts forward his heliocentric theory and names "gravity" the power that moves the celestial bodies.

That is how Copernicus displaced the geocentric and in extension, the anthropocentric system and put the sun in the centre of the universe.

What a reversal!!! These ideas could not, of course, run with any objections and rejections! The Christian and the Protestant world could not accept but only the geocentric and anthropocentric world.

Copernicus though, described the laws of nature and Kepler completed it, (he formulated the laws of the motion of the planets).

Galileo then formulated the first laws of mechanics and contributed to the turn of science for observation, experiment and mathematical calculation.

Finally, this entire journey would reach Newton's formation of a mechanistic cosmic idol of nature with solid and undivided atoms, forces from distance, the Euclidean and infinite space.

After 2000 years of human history, Aristarchus, Copernicus, Kepler and Galileo, the heliocentric system had finally been proved.

TOWARDS THE 19TH CENTURY

Since the 19th century the development of science and technology has been rapid. Albert Einstein and Relativity Theory (it's about a physical phenomenon description).

Quantum Theory (it's the theory of light, the idea that light exists as tiny packets, or particles, which he called photons)

"BigBang" theory (it's the leading explanation about how the universe began) which we are going to see later on, in this presentation.

All these are some of the theories and speculations formed that period of time. Newton's theories on gravity effects were determinant in the development of mechanics and cosmology. Newton speculated that gravitational interaction is transmitted within space at an infinite speed.

Now, any quest for the creation of the world is based on positivistic and rationalistic perceptions.

Hume's and Kant's theories are such, accordingly.

For Hume: an inexhaustible source of knowledge is experience.

For Kant: all matter was at first as gas, spread more or less same in the universe.

Today we know that laws of nature are not eternal but just "instant" approaches to reality, which they do not exhaust.

THE SCIENTIFIC TRUTH OF TODAY

In 1924, Hubble established the Andromeda nebula as a galaxy just the same as ours.

This establishment opened new horizons to cosmology and the evolution and structure of an approachable part of the universe.

Mathematical frame and microphysics in cooperation

As we said before, with Einstein, the mathematical frame for cosmological models existed, but microphysics completed the large-scale undertaking.

Their dialectical combination led to contemporary cosmological models.

So, their dialectical combination brought about the cosmological models we are going to see right away.

CONTEMPORARY COSMOLOGICAL MODELS

- Expanding universe model (Alexander Friedman, 1922)

A pioneering theory that the universe was expanding, governed by a set of equations.

- Steady state universe model (1948, Hermann Bondi, Thomas Gold and Fred Hoyle)

The steady state model is based on the assumption that on the large scales the universe is completely homogenous.
The density of matter in the expanding universe remains unchanged due to a continuous creation of matter. That is, the observable universe is basically the same at any time as well as at any place.

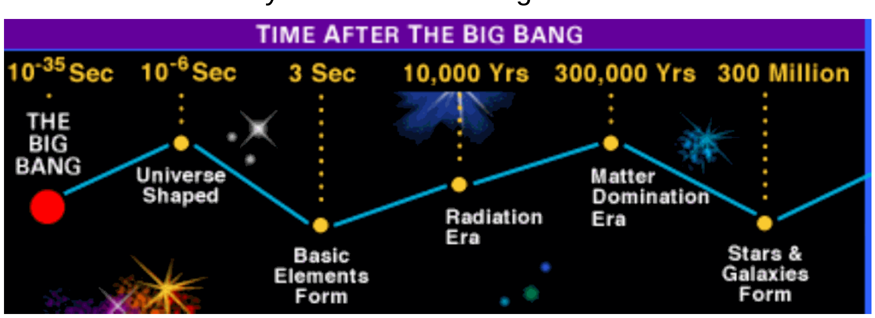
- Big Bang model (Georges Lemaître, 1931 – model advocated and developed by George Gamow, 1952)

- Big Rip theory (2003 - Marcelo Disconzi)

At the time the Big Rip occurs, even space time itself will be ripped apart and the scale factor will be infinity. That is the end of the universe based on its increasing expansion rate.

FOCUS ON THE PREVAILING THEORY OF BIG BANG – A BRIEF TIMELINE

As we can see, the big bang starts at 10 to minus the 35th power of a second. The universe is shaped at 10 to minus the 6th power of a second and at 3 seconds we have the basic elements formed. After 10.000 years comes the Radiation Era and Matter dominates after 300 thousand years. Finally, it needs 300 million years for stars and galaxies to form.



Conclusion

Man in need of answers on cosmogeny. To sum up, the following brief chronicle can show us the great leaps forward
1791: Erasmus Darwin: The description of a universe that expanded and contracted in a cyclic manner was first put forward in a poem.
1848: Edgar Allan Poe: presented a similar cyclic system in his essay titled "Eureka: A Prose Poem"

1927: Georges Lemaître: proposed an expanding model for the universe to explain the observed redshifts of spiral nebulae.
1929: Hubble: discovered that, relative to the Earth and all other observed bodies, galaxies are receding in every direction at velocities (calculated from their observed red-shifts) directly proportional to their distance from the Earth and each other.
1931: Lemaître: proposed in his "hypothèse de l'atome primitif" (hypothesis of the primeval atom) that the universe began with the "explosion" of the "primeval atom" — what was later called the Big Bang.

The Big Bang theory could explain both the formation and the observed abundances of hydrogen and helium.
1990s and the early 21st century:

Huge advances in Big Bang cosmology were made as a result of major advances in telescope technology in combination with large amounts of satellite data, such as COBE (COpenhagen and BErlin – the two founders' origin), the Hubble Space Telescope and WMAP
(Wilkinson Microwave Anisotropy Probe)
That is how we can understand how galaxies form in the context of the Big Bang and we can also understand what happened in the earliest times after the Big Bang and reconcile observations with the basic theory.

BUT:

- Actually, what existed before the explosion?
- What is that dark energy in the universe?
- Will a similar explosion happen again?
- Is there an end somewhere?
- Is there going to be a total disaster and start from scratch, all over again?
- Is there someone, something protecting man kind and life, up there?

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A QUANTUM BRAIN?

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ABSTRACT

The inconsistencies of Bohr's model, Schrodinger's equation and Heisenberg's uncertainty principle contribute to the evolution of the quantum mechanical model. Many concepts of this model (e.g., particle-wave duality, superposition) are counter-intuitive. But is this due to our cognitive inability or to inherent obstacle in nature? Abbott with his famous "Flatland" would argue in favor of the first whereas Heisenberg for the second. Interestingly, many researchers propose that quantum processes may also happen in the brain. Quantum entanglement is a phenomenon that might underlie neural processes. If this holds true, then could it also be true that the brain can engage in two or more thoughts at the same time by quantum superposition? Moreover, could it be true that quantum computers might lead to human-like intelligence in computers?

BOHR'S MODEL OF THE ATOM

Danish physicist Niels Bohr won the Nobel prize in Physics in 1922 for his work that founded his model for the atom. Bohr's model can be described with the following statements:
a) Electrons circle the nucleus in certain orbits.
b) The number of the orbits in an atom is finite.
c) Electrons can move from one orbit to another.
d) The energy of the orbits is quantized.
e) Each orbit is of certain energy, meaning, that the lowest energy is found in the smallest and closest to the nucleus orbit, while the highest energy is found in the bigger and most distant from the nucleus orbit. Each orbit is characterized with a number n, (ex: n=1, 2, 3 etc.) called the principle quantum number.
Although Bohr had proposed a good model of the atomic structure, used even today for educational reasons, it was later proved that it fails to explain the real atomic structure. For example, Bohr's atomic model only works for atoms with one electron (hydrogen), as his calculations do not include repulsions between electrons. Moreover, the Bohr model is incompatible with Heisenberg's Uncertainty principle. This happens because the Bohr model stated that in the atom we know both the location and the energy of an electron simultaneously. However, according to the Uncertainty principle we can't know both exactly the position and the momentum at the same time.

De BROGLIE HYPOTHESIS AND EXPERIMENTAL PROOF

In 1924, Louis-Victor-Pierre-Raymond de Broglie, influenced by Einstein's papers published about 20 years before (in 1905) concerning the behavior of electromagnetic radiation as particles, stated a theory which gave him the Nobel Prize for physics in 1929. he was nominated for "the discovery of the wave nature of the electron". De Broglie suggested that all matter has wave properties, a concept known as "de Broglie hypothesis".
His revolutionary idea, which is, nowadays scientifically proved, through several experiments, with widely known the once performed by C. J. Davisson and L. H. Germer.
The Davisson-Germer experiment was conducted in 1923-1927 and their goal was to study the bouncing of electrons when they were fired by an electron gun towards a nickel crystal in a vacuum chamber. In order to detect the bounced electrons, a moveable detector was used. When an accident happened during the experiment, air was introduced in the vacuum chamber and a film of nickel oxide was formed on the surface of the metal. When the two physicists heated the metal to remove the oxygen, the nickel atoms were rearranged, but the scientists did not know it. When the experiment started again, electrons were diffracted from atoms that were also inside the nickel lattice. At certain angles, the detector resulted in peak intensity which could be explained by the wave behavior of electrons. These intensity peaks were the constructive interference of electron diffraction. The hypothesized de Broglie wavelength of the electrons was expected to be 0,167 nm. The measured wavelength through Davisson-Germer experiment was 0,165 nm according to Bragg's Law (i.e., Bragg's Law correlates the wavelength of a wave (λ), the distances (d) of particles in a crystal lattice and the scattering angle, θ). Two waves can interact in a constructive manner (they are added) or in a deconstructive manner (they cancel each other).

SCHROEDINGER'S EQUATION AND HIS CAT

Schrodinger equation is extremely important in quantum physics and governs many phenomena of the microscopic world. Erwin Schrodinger was a physicist who set out to find a wave equation to describe the electron of a hydrogen atom (1926).
Schrödinger applied it to the hydrogen atom, predicting many of its properties with remarkable accuracy and by that, he established the correctness of his equation.
In his "thought experiment", a cat was placed in a closed box with some certain objects like a radioactive source, a Geiger counter to measure radioactivity and a bottle of poison.
If radioactivity is detected, the Geiger counter will trigger a hammer to break the poisonous bottle and the cat will be killed.
This experiment was aimed to reveal the flaws of the 'Copenhagen interpretation' of quantum mechanics. This interpretation supports that a particle exists in all states at once until it is observed.
In similar context, the cat too is both alive and dead until the box is opened. Schrödinger actually tried to use common logic that says this can't happen in order to emphasize the limits of the Copenhagen interpretation when applied to practical positions.
But as irrational as it may seem, Schrodinger's cat is actually real!

HEISENBERG'S UNCERTAINTY PRINCIPLE

Heisenberg's Uncertainty Principle was firstly discovered by the German physicist Werner Heisenberg back in 1927. With this principle Heisenberg won the Nobel prize in 1932. According to this principle, humans cannot define the exact position (x) and the exact momentum (p) of a particle at one specific time. This does not happen because we are unable to define these two phenomena, but it is an inherent principle in our world. The mathematical formula for Heisenberg's principle of uncertainty is $\Delta x \cdot \Delta p \geq h/2\pi$. Heisenberg said, that when we measure the position of a particle with uncertainty (Δx) and at the same time we measure its momentum (Δp), again with uncertainty, then the product of these two quantities can't be less than a specific number ($h/2\pi$). Having said that, the more precisely the position of some particle is determined, the less precisely its momentum can be known and vice versa. The uncertainty principle is one of the most famous aspects of quantum mechanics and through this principle we can see how quantum mechanics (physics in the microcosm) differ from the classical theories (physics in the macroworld).

QUANTUM SUPERPOSITION AND QUANTUM ENTANGLEMENT

Quantum superposition and quantum entanglement are two of the most fundamental concepts of quantum physics. Quantum physics as a theory was born from the need to explain physical phenomena classical mechanics could not explain rationally, through classical laws.

In quantum mechanics, a particle is simultaneously in all possible places its wavefunction allows it to be, it is at the same time in multiple states. This is called "superposition" of the particle, and we say that the particle is in a "superposition of states". Only by observing the particle, it is forced to pick a specific place to be in, or a specific state, and this picking is random, within the constraints of the wavefunction. With observation we create reality, and we move from quantum to classical realms.

Next, another notion of quantum physics, closely related to the superposition idea, is the one of quantum entanglement. Quantum entanglement is a quantum phenomenon in which the quantum states of two or more objects in a system have to be described with a reference to each other, even though the individual objects may be spatially separated. In simplistic words, we could imagine quantum entanglement this way: if a system S consists of two particles A and B, which have once interfered with one another, then connections of a non-classical character are developed between them, and the quantum state of one can influence that of the other. So, if both particles are in a superposition of states, and one of them is detected, and acquires a specific state, the other particle, no matter how far away, will always acquire the other, opposite state. This principle suggests that events happening in one place could influence other events happening someplace very far away.

WAVE-PARTICLE DUALITY

Albert Einstein was the physicist, who showed in 1905 that electromagnetic radiation consists of particles of energy and thus should not only be considered as a form of electromagnetic waves, but also as particle-like.
In his independent experiments, Arthur Holly Compton discovered the Compton Effect and also proposed that light is a particle.
On the other hand, Thomas Young back in 1801 had demonstrated through light diffraction experiments that light is a wave.
However, these two characteristics, that seem totally different theories, are related to each other. In fact, both of them co-exist and hence every type of wave should present a particle character and vice versa. This idea contributed to the discovery quantum mechanics and was a revolutionary idea in the field of physics. Schrödinger gave a mathematical form to Broglie's hypothesis, referring to the wave behavior of particles, which later became a popular applicable equation in quantum mechanics.

QUANTUM COMPUTERS

Nowadays, many of our daily problems such as problems in mathematics, in economics, problems concerning public transportation, in modern and robotic medicine are solved easily by using computers. Classical computers use memory registers in order to store data, which is expressed with a sequence of binary digits (bits). The process of these data drive to the solution of a problem. Classical computers need to be more powerful in order to manipulate large amounts of data and solve more demanding and of higher complexity problems. For instance, a problem of this kind could be the management of data which is transferred through the worldwide web when a number of three billion users are connected at the same time. In order to solve problems like these, classical computers need to use very efficient processors.
The same problem could be solved more easily if we use quantum computers. Quantum computers transform data in the form of qubit. A bit can exist in two states, namely "0" and "1". A qubit can follow the principle of superposition and exist as "0" and "1" at the same time (i.e., similar to Schrodinger's cat!).
What quantum computers can do in the present is to function faster the classical computers and distribute larger amounts of data. The first attempt in the making of quantum computers was a few years ago.
There are many different ideas and methods we could follow in order to develop a quantum computer.
One idea is based in "Nuclear Magnetic Resonance (NMR)". The theory is based on the collection of a large number of chloroform molecules in a water tank, at room temperature. An external magnetic field is applied to the chloroform molecules. As a result, the nuclear spin in the molecules could be of the same direction with the external magnetic field or of the opposite direction. A nucleus with spin in the same direction of the external magnetic field is interpreted as "1" and a nucleus with spin in the opposite direction is interpreted as "0". When we use electromagnetic pulses to the chloroform molecules we cause spin states to flip. Thus, we create superposition phenomenon with parallel and anti-parallel states of spin. With the superposition phenomenon we can succeed not 2-qubit but also 10-qubit or even 15-qubit.

FLATLAND

In 1884, Edwin A. Abbott wrote a stunning piece of work, a book which is called “Flatland” and since then it has been striking a strong chord with readers everywhere. This book is known as “A romance of many dimensions”.
This coming-of-age romance introduces the readers into another ultimate world, the world of two dimensions which is called Flatland. In other words, this world can be easily imagined as a huge piece of paper on which many shapes such as lines and hexagons live, but they are unable to rise above this. To put it another way, they can only move forward, backward, leftward or rightward, but no upward or downward.
What will happen when a three-dimensional object (e.g., a cube) enters the two-dimensional world? Well, the two-dimensional organisms will observe their intersection. If the cube touches the 2D-world only with one vertice, then the 2D organisms will observe a dot. If the cube touches the 2D-world with one edge, then it will be observed as a line. If the cube is merged in the 2D-world, it will be observed as a rectangle.
In a similar context, we, humans, understand only three dimensions and live in a 3D-world. Is it then possible that we can observe the same object (e.g., an electron) in different ways (e.g., wave or particle), although the true nature of the object is one? Abbot would definitely argue in favor of this idea.

QUANTUM COGNITION

Quantum mechanics (QMs) is unanimously fundamental in physics and chemistry. Recently, the role of QMs in biological phenomena is explored. Some researchers propose that QMs may play a role in brain activity.
“A neuron is a nerve cell that receives, processes, and transmits information through electrical and chemical signals.”
Synapses are the specialized connections between the end of one neuron (i.e., axon terminal) and the “start” of another neuron (i.e., dendrite). Chemical substances called “neurotransmitters” mediate the propagation of the signal in synapses.
Physicist Mathew Fischer published a study in 2015 that claims that quantum phenomena like entanglement might occur in the brain.
Fischer reports a study from 1985, where lithium drugs, known for treatment of mental illnesses, where used in experiments with rats. Researchers used two different isotopes of lithium. Pure lithium-6 had the exactly opposite result in the behavior of mother-rats when compared with pure lithium-7. These two isotopes differ in the number of nucleons that they contain. Lithium-6 has 3 protons and 3 neutrons, whereas lithium-7 has one additional neutron. Nucleons produce a magnetic field and “act like little magnets” which can be described by a concept called “nuclear spin”. The spin can be “up” or “down” (i.e., opposite orientation of the magnetic field).
Atoms like phosphorus are potential candidates that can act like lithium-7. Phosphorus nuclei have odd number of nucleons which makes the nuclei behave like “little magnets”. When two phosphorus atoms are entangled, then the spin of one atom is correlated to the spin of another. Phosphorus is ubiquitous in human cells. Adenosine triphosphate (ATP) has three phosphate groups (PO_4^{3-}). Pyrophosphatase ($P_2O_7^{4-}$) contains two phosphate groups that are entangled.

Phosphate ions (PO_4^{3-}) can aggregate with calcium ions to form a "Posner cluster". Two Posner clusters can make a Posner dimer. This dimer is proposed to enter into neurons.
Fischer hypotheses that when Posner molecules are hydrolyzed, then they can release calcium and trigger the firing of a signal from a neuron to another. With entangled Posner molecules, the signal firing might also be entangled. This process is confirmed in vitro.

HUMAN BRAIN VERSUS COMPUTERS

The human brain presents similar functions to a computer since, both are used for storage, processing information and execute tasks.
Computers have certain primary purposes: Entering data, manipulating data, viewing processed data and storing data. They can transform raw data into information and operate under the control of instructions stored in its own memory unit.
Brain and computer both work with electrical signals. As it concerns the memory, computers can continue to store memories as they add more RAM and stores them in a more orderly way than the human brain.
However, computers can only follow instructions made by a programmer and act under logic, while the brain seem to behave freely, using emotions, reasoning and common sense, without having something to guide it. Moreover, the human brain can easily adapt to new circumstances and learn faster. Complex computer processes can be achieved by a few hundred neuron transmissions, requiring much less energy and performing at a greater efficiency.
One of the things that truly makes the brain stand out is the flexibility that it displays. The human brain has the ability to rewire itself (neuroplasticity). Neurons have the ability to disconnect and reconnect with others, and even change their basic features, something that even a carefully constructed computer can't do.
Interestingly, the fastest supercomputers created, which are massive, haven't even reached the human brain's processing speed, yet.

ARTIFICIAL INTELLIGENCE

Artificial Intelligence, also known as AI, could be considered as a science that combines computer science and engineering. In fact, computer programs and algorithms are applied in machines providing them with human intelligence that enables them to react and to work as human beings. More precisely, these machines, also known as robots, have affective computing systems, which enable them to detect human emotions and reply appropriately.
Nevertheless, the use of algorithms is one of the most important characteristics of an artificial intelligence machine. In fact, algorithms allow the intelligent machines to work on information and to make calculations step by step. In that way, robots are able to reach conclusions and provide themselves with knowledge that they created by themselves.
Furthermore, these machines have the ability of solving complex signal processing and pattern recognition problems by using the ANN (artificial neural networks). These networks "learn" to perform tasks without being programmed so. For example, image recognition is a system that can learn to identify a dog in a picture, if the system is provided with random images that contain dogs.

Moreover, a really important characteristic in AI are the heuristics which function when someone makes a question or gives an order for which the response needs to be immediate, but there is not an algorithm for it, so the machine does an approximation so that it can fill the gap with the most proper way.
In fact, all these functions characterize an artificial intelligence because of all these quantum machines that have been applied and are still being applied into computers. It is thought that the artificial intelligence is a scientific invention that will soon resemble a lot more the human intelligence. In fact, it is believed that virtual scientists would be able to look deeply into scientific issues. Also, they would be able to create more powerful computer programs and they could come up with new theorems for mathematics and for our universe. Consequently, quantum machines contribute greatly to the progress of our world.

CONCLUSION

In quantum mechanics, many counter-intuitive concepts exist. Quantum superposition and entanglement is a reality in quantum computers. Researchers propose that these phenomena may also occur in biological procedures (e.g., neuron signal firing). If this holds true, then could it also be true that the brain can engage in two or more thoughts at the same time? Moreover, quantum phenomena in computers exponentially increase their processing capacity. Could it also be true that quantum computers bring us a step closer to human-like intelligence in computers?

ENLIGHTENING LIGHT: YOUNG'S DOUBLE SLIT EXPERIMENT

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ABSTRACT

Young's Double Slit experiment, firstly implemented to scrutinize the true nature of light, a question which had created contradistinctions between physicists everywhere, undoubtedly proved its wave nature. We present his experiment, and explain the interference pattern we saw as its result. We also demonstrate its latest version, using single electrons, and explain single-particle interference according to the Copenhagen and Many Worlds Interpretations. We analyze fundamental notions of quantum physics, such as the wavefunction, superposition, complementarity, and multiverse. We conclude that this experiment was a breakthrough, establishing the radical theory of light's wave nature. Extending, we reflect on the boundaries between classic and quantum realms. Finally, we acknowledge the need to broaden our already formulated perspective of the world, in order to understand quantum physics.
Keywords: Double-slit experiment, quantum physics, wave interference, light's nature.

"Perhaps the light will prove another tyranny.
Who knows what new things it will expose."
C. Cavafy

INTRODUCTION

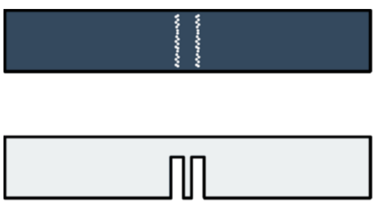
The light entering through the closed windows will lead our great poet to an existential apocalypse. An apocalypse, however, was also the unveiling of light's dual nature. Perhaps one of the most famous experiments conducted for the sake of understanding the behavior of light, particles, matter and radiation, was the optics Double Slit Experiment, which sought out to scrutinize and determine the true nature of light. It began as a thought experiment, and was implemented by the British physicist Thomas Young in the early 19th century.

THE SCIENTIFIC CONTRADISTINCTION ABOUT LIGHT'S NATURE

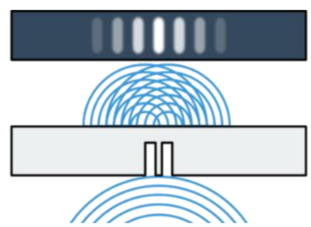
Up until that time, a major contradistinction was going on between physicists all around the world, concerning light's nature, but also light's speed. For the second question, scientists have agreed that light travels with extremely fast, but not infinite, speed. For the first question however, matters have not been quite as simple. There were two theories that physicists advocated: Some, including Newton, supported the view that light was made of particles, acquiring that thesis based on optical phenomena such as light's reflection and refraction, while others, mainly represented by Hooke and Huygens pledged differently, and endorsed the notion that light was a wave, which would explain light diffraction. Young's intention was to initially study this phenomenon of light diffraction, but through his experiment, it became discernible that light undoubtedly has a wave nature.

THE EXPERIMENT WITH ORDINARY THINGS

Before we delve into the experiment itself and its explanation, let's try to reenact it with other things instead of light, in order to understand it better.
Imagine having a board with two narrow slits, and another screen some distance behind it. If we start throwing pebbles at the first board, some of them will bounce off, and some will get through the slits, creating two separate rows of hits, one behind each slit. (Fig.1). If, though, we send a wave of water to the board, while it passes through the two slits, it will divide into two new waves, which will interfere, and create an interference pattern on the screen. (Fig.2)
These examples vividly show how localized particles would act, in opposition to waves.



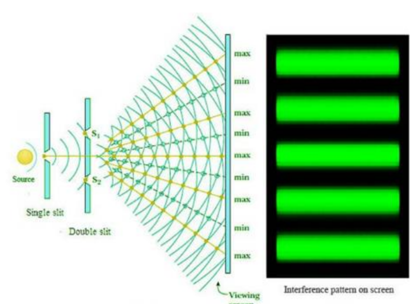
(**Fig.1.** From: http://www.kathimerinifysiki.gr/2017/06/ti-einai-kvantiki-fysiki.html)



(**Fig.2.** From: http://www.kathimerinifysiki.gr/2017/06/ti-einai-kvantiki-fysiki.html)

YOUNG'S EXPERIMENT WITH LIGHT

Now, in the place of the pebbles or the wave of water, Young shone light to the board with the slits. On the screen behind the board an interference pattern was created, with bright and dark stripes. This was pure evidence that light had passed through the slits as a wave, then was split into two separate waves passing through each slit, which interfered with each other, eventually creating the interference pattern on the screen. (Fig.3)



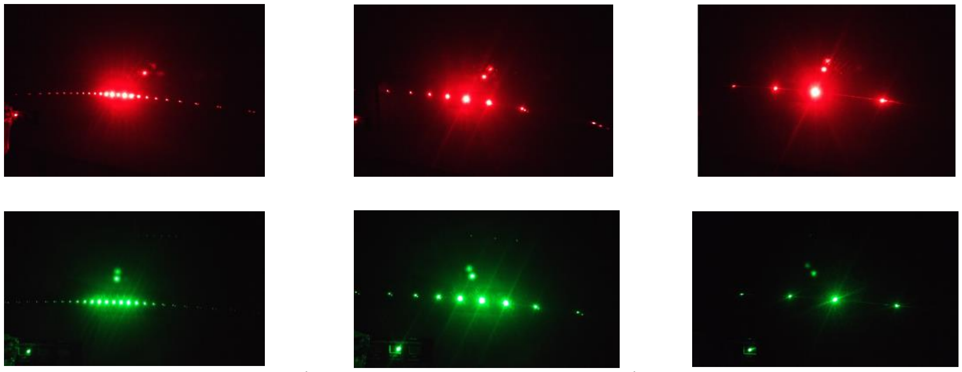
(**Fig.3.** From: https://practicallawandjustice.liberty.me/the-double-slit-experiment-a-rational-explanation/)

The bright stripes were the result of the two waves of light arriving in phase, which means that each crest of one wave overlapped with a crest of the other one, and the same goes for their troughs. The interference, in this case, is called constructive interference. Oppositely, the dark stripes were the result of the two waves arriving on the screen out of phase, meaning that the crests of one wave overlapped with the troughs of the other, canceling each other out. This is called destructive interference.

But how exactly is the constructive and destructive interference produced? The wave that arrives at the double slit splits into two new waves, each of them leaving one slit. These two waves, as they leave the two slits, are in phase, since they were produced at the same moment, from the same source, the initial wave. For the central bright point we could say that the two waves interfered constructively, and, more specifically, since they are in phase, they travelled the exact same distance to the screen. The next bright line would have to be the result of constructive interference too. But in this case, the two waves would not travel the same distance to the screen; one of them would travel less, and the other more. But how much exactly would that extra distance be? Well, since the waves leave the slits in phase, and we need the peaks and the troughs of the waves overlapping for constructive interference, the extra distance would be one more wavelength. For the next bright spot, we will apply the same way of thinking, and we would see that the difference between the distances the two waves travelled would be two more wavelengths. We call this difference path-length difference (PD). We come to the conclusion that if the waves leave the source in phase, the points where we observe constructive interference are the points where the path length difference takes values of an integer times the wavelength (1λ, 2λ, 3λ… $m\lambda$). The exact type for this is $PD= m\lambda$, where $m \in \mathbb{Z}$.

The opposite happens when we observe destructive interference. The two waves, as mentioned, leave the slits in phase. If the bright bands are the result of integer times the wavelength, and the dark bands are exactly inbetween the bright ones, then this would have to mean that the light travelled half as much as it did to reach the next constructive point. This would have to mean that it travelled a length which is a half-integer multiple of the wavelength. Indeed, for the waves to interfere destructively, we would need a peak overlapping a valley. To achieve that, the path length difference of the waves, which were initially in phase, would have to be half as much as previously, to have the peaks overlapping the valleys. So in this case, $PD= m\lambda$, where $m= 0.5, 1.5, 2.5…$

We conducted the experiment ourselves in Pierce College, using red and green laser and diffraction gratings of 100, 300 and 600 slits. We saw from the results that in both cases, as we used diffraction gratings with more slits, the maximum points moved farther away. To understand this, let's imagine two slits instead. If we have a larger distance between the slits, the lines that would lead to the bright points are closer, and so are the points. Oppositely, if we decrease the distance between the slits, the lines of constructive interference move farther away, along with the maximum points. So, the more slits in the same space, the farther the constructive points would be. Furthermore, we observed that with green laser, the maximums appeared slightly closer. This is because green light, compared to red, has a smaller wavelength.



(Fig.4: Taken at Pierce's laboratory)

MODERN REPETITION OF YOUNG'S EXPERIMENT WITH ELECTRONS

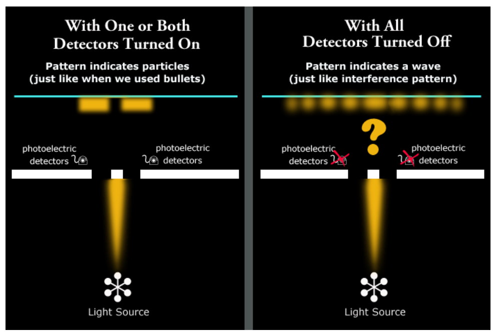
Over the years, the same experiment has been repeated many times, with different particles and under different conditions. The initial experiment may have shown how photons exhibit light's wave nature, but as physics has progressed over time, and the new field of Quantum Physics has emerged, it has been proven that photons are not the only particles that have a wave-like nature, but, in fact, all fundamental particles do, and even larger molecules!
Let's take, for example, the case of electrons. Their wave-like nature can be vividly illustrated through the phenomenon of electron diffraction, which was studied initially by Louis de Broglie (1924), who was aiming to prove the dual nature of other particles as well, apart from photons. If we fire electrons to a thin metal foil, and place a fluorescent screen behind it, we would see that on the screen there would be a diffraction pattern with concentric circles. These interference effects owe to the wavelike nature of the beam of electrons when passing near matter (According to de Broglie's proposal, electrons and other particles have wavelengths that are inversely proportional to their momentum. Consequently, high-speed electrons have short wavelengths, a range of which are comparable to the spacings between atomic layers in crystals).
As a more modern repetition of Young's experiment, extreme interest shows the case in which we throw single electrons to the double-slit board, one at a time, in time intervals which allow each electron to leave the source and reach the apparatus, which we use nowadays in these experiments, instead of the simple screen. This would remind us of the previous example with the pebbles. The electrons, as we consider them localized particles, would have to go either through the one slit or the other, and end up in places right behind each slit, depositing all their energy in one place, each one of them acting individually and independently from the rest. However, the interference pattern is observed once more, this time as a result of many accumulated electrons. But how is this possible? The explanation is closely related to the fact that electrons have a wave nature as well.

QUANTUM THEORY AND TWO INTERPRETATIONS OF THE EXPERIMENT

This is where the quantum theory gets involved into this mind-boggling experiment. Quantum Mechanics is one of the two pillars of modern physics, the other being Einstein's Theory of Relativity. Quantum theory is the most contemporary theory of physics, and describes the behavior of microscopic particles, such as protons, electrons, and photons. The term "quantum"

derives from Latin, and means “how much”. For example, the electron is a quantum of negative charge, meaning it’s the smallest quantity of negative charge there can be. From this directly derives the fact that quantum physics is discrete. This means that the energy contained in a quantum field comes in integer multiples of some fundamental discrete energy amounts.
To return to our experiment, quantum physics has been able to give explanations to its results, however convoluted and counter-intuitive they might appear.
According to the Copenhagen Interpretation of quantum physics, mainly proposed by Neils Bohr and Werner Heisenberg, a particle in the double slit experiment exists only as a wave of possible positions that includes all its possible paths. The possibilities that we find the electron in a specific place constitute its wavefunction. Furthermore, this theory suggests that there is no point in discussing where the particle is until we detect it. It is only when the particle is observed that the wavefunction collapses and the position of the electron is finally decided.
It looks as though the universe allows all possible paths of the electron to happen, but choses which one actually took place the last moment. And, furthermore, all those possibilities interfere, and hence produce the result seen on the screen. The particle simultaneously takes all possible paths, which means it passes through both slits. It attempts all possible trajectories, which interact with one another, to produce the final outcome, the interference pattern. This means that the electron is in a state of superposition. Superposition is the ability of a quantum system to be in multiple states at the same time until it is measured. Each possible position of the electron constitutes one possible state for it to be in. So we say now that the electron is in a “superposition of states”, since we theorize that it is not in a specific place, but in all places where its wavefunction allows it to be. According to the Copenhagen Interpretation, only measurement collapses the possibility space of the wavefunction, and stimulates the choosing of a specific reality. The collapse of the wavefunction signifies the transition between quantum and classical realms, since we stop talking about quantum states of superposition, and we observe a single, specific reality. Moreover, in this interpretation, the final choice of the experiment is non-deterministic, meaning that it is random, within the constraints of the wavefunction. In this case, the interference is not between . electrons, but rather that each electron interferes only with itself due to the quantum uncertainty of which path through the optical apparatus it takes. What we observe as an interference pattern is the accumulated sum of many single particle interference events (Rueckner & Peidle 2013). Therefore, the Copenhagen Interpretation suggests that there is no specific reality. Reality is created by observation.
However, there is another way to explain this phenomenon. The Many Worlds Interpretation’s, main idea is that the wavefunction never collapses. It was proposed by Hugh Everett in 1957. It proposes that when a different path occurs in a situation, such as in our experiment, the universe splits into more universes, each one allowing one possible reality to happen. The many universes combined together form the multiverse. All the different possible histories, all the possible trajectories of the electron continue existing, one in each separate universe. The many separate universes somehow interfere, producing the result on the apparatus. By observing the position of the electron, the interference of the different universes ceases to exist, so the interference pattern disappears. Unlike the Copenhagen interpretation, the Many Worlds Interpretation is a deterministic one. And, oppositely to the Copenhagen Interpretation, the Many Worlds Interpretation does not dismiss the notion of existing realities. However, it suggests that

instead of one, there are infinite realities, each one taking place in one of the infinite separate universes of the multiverse, and we just happen to be in one of these universes.
Nevertheless, whichever interpretation one choses to believe, there is one last mystery in the modern repetitions of the experiment. Let's assume that we are conducting the experiment firing single electrons to the slits, placing a detector next to them. The detector will observe the electrons passing through the first slit. We assume that the ones it doesn't detect have passed through the second slit. But what we see this time is a different result than what we have been observing and trying to explain up until now. What we see is two separate stripes of detected electrons behind each slit, just as we expected them to act in the beginning, exposing their properties as localized particles. So what is going on? How do the particles know when they are being observed and when not? Is the universe inconsistent with itself? This is one excellent example that vividly portrays the mystery of quantum mechanics.



(Fig.5. From:https://physics.stackexchange.com/questions/376336/double-slit-experiment-what-effect-does-the-detector-actually-cause)

If we try to explain this phenomenon according to the Copenhagen Interpretation, our way of thinking would be the following: As mentioned, this interpretation suggests that with observation we create reality, and find the electron in one place. So, when the detector observes the electrons, they stop acting according to the properties that their wavefunction gives them, and they find themselves in one specific place! Thus, this does not allow single-particle interference, since the electrons do not exist in multiple states at once, in order to have the chance to interfere with themselves. They now act like localized particles, and pass through one slit or the other.
If, on the other hand, we adopt the notions of the Many Worlds Interpretation, the explanation would argue on the following lines: in order to have the interference pattern as a result of the experiment, it was suggested that the electron exists in multiple universes, and these multiple universes interfere and produce the result. But, by observing the position of the electron, we stop this merging of realities, we disallow universes to interfere with one another, and we focus on one single universe, where the electron exists in one specific place, in the position where we observed it. So now, the electron is one single particle, and has no way to interfere with other versions of itself. Thus, we can say again that it passes through one or the other slit, and ends up in a specific place on the apparatus.

CONCLUSIONS

Through the puzzling and obscure Double Slit experiment, one of the biggest breakthroughs in physics was discovered: light's wave nature. However, taking in mind the photoelectric effect, which proves the particle-like nature of light, it becomes evident that light has a dual nature, acting both as particles and as a wave. The fact that wave and particle natures “coexist”, is what Bohr called “complementarity” (Haroche & Raimond, 2006: 11-12). This was extremely important, since it gave an undeniable answer to the argument about light’s nature, revealing clearly its also being a wave. It became, therefore, the basis for later theories and further scientific research on light.

This experiment, especially in its modern variations, gives us a glimpse of what is going on in the quantum world. It reveals the wave nature of all particles. This notion is also one of the fundamental principles of quantum mechanics, suggesting that everything in the universe has both particle and wave nature. Moreover, since this experiment has also been conducted with atoms and large molecules, and has given the same results as when conducted with smaller particles, it is ideal for the exhibition of the boundary between the microscopic world, explained by quantum theory, and the macroscopic, explained by classical physics. By gradually increasing the size of the particles in our experiments, we might hope to explore the frontier between quantum and classical physics. So, obviously, the question is also provoked: “Why do macroscopic objects made of particles which, individually, obey quantum rules, behave classically?” (Haroche& Raimond, 2006:68). Anyhow, quantum mechanics is an “uncommon-sensy theory” (Feynman, 1985:5), which provokes the interest of the researchers, and laymen as well, but still is ambiguous and, many times, incomprehensible.

After, however, we have comprehended in depth the philosophy of the Many Worlds Interpretation, we see that many philosophical questions arise, as to how we, as people, experience and understand reality. This interpretation suggests that there are infinite universes, each one hosting a different reality. So now, we might contemplate that there are actually infinite versions of us out there, each one experiencing a wholly different reality. We are just one of our infinite versions, who, in this universe live this specific life, while in another parallel universe we could be entirely different people! So, could we say that our actions are really results of our own free will? Are we creatures with the chance to formulate our own life as we want it, and take our own initiatives, or is our path in life predestined by the multiverse? Are we just beings that fulfill the universe’s where we live in purposes?

But all these profound questions could be further discussed and… enlightened not only in another conference, but even perhaps in another… universe!

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STRUCTURE, PROPERTIES AND FUNCTION; THE QUEST FOR MATTER'S MINIMAL PARTS

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ABSTRACT

The Standard Model of Particle Physics represents the current state of Humanity's age old quest for matter's minimal parts. In spite of its success it displays a feature that is common to the early stages of almost all scientific fields. In their early stages scientific fields go through a phase of classification of the various elements that they study which is based on certain conserved properties of the elements within each class. For example, the classes "Genus " in Biology or "Periods" in Chemistry. As the fields progress the basis of these properties and classification is understood as being due to the relatively stable combination of certain entities that constitute the minimal parts of matter specific to each field. These minimal parts would be "Nucleotides" for Biology, "Nuclei and Electrons" for Chemistry etc. If we break up these structures we pass from one scientific field to another, from Biology to Biochemistry to Chemistry to Physics. In this sequence the energy required to break up the structure at each level is smaller than the mass of the constituents involved at that level. However, if we want to explore even smaller scales, Heisenberg's uncertainty principle implies that the energy required to achieve this becomes equal or even greater than the mass of the constituents. In addition, the constituents' masses become smaller than the energy that binds them together.
Have we therefore reached the end to this sequence? Are the minimal parts of the so called "Standard Model" of the world i.e. Quarks and Leptons truly "fundamental" or "elementary" or should we expect another level of structure on an even smaller scale? Does it make sense to think in terms of structure and constituents beyond a certain scale?

INTRODUCTION

In this presentation, addressed to high school students, we will try to outline the centuries' old quest for the nature of matter's properties in terms of constituents, from Demokritos' "atoms" to today's Standard Model "particles" and to point to what may lie beyond. A key feature of all these searches involves the distinction between "intrinsic" and "structural" properties. An intrinsic property is one that an object possesses independently of being alone in the world whereas a "structural" property is one that is due to the internal organization of certain smaller constituents. Another key feature is that of a "conserved" property, that is a property that persists when an object undergoes transformations. A conserved property can either be intrinsic or can be structural, in which case it disappears if this structure is altered.
Conserved Properties form the basis of classification schemes in different branches of Science . This classification (Taxonomy) characterizes the early stage of every science and is based on phenomenology, that is on the apparent behaviour of the objects under study.

A brief survey of "conserved" properties in different scientific fields and their subsequent understanding in terms of underlying structures will allow us to discuss certain analogous features of today's Standard Model of Elementary Particles. These features may be pointing to future developments beyond our present understanding of our world.

BIOLOGY-CHEMISTRY-ATOMIC PHYSICS-NUCLEAR PHYSICS

Taxonomy in Biology. from Ancient Greek -τάξις (-taxis), meaning "arrangement"/"ordering"", and -νόμος (-nomos), meaning "rule+", is the process of defining and naming groups of biological organisms on the basis of certain shared and conserved characteristics and conserved properties. Figures 1, and 2. illustrate this classification process in plant biology.

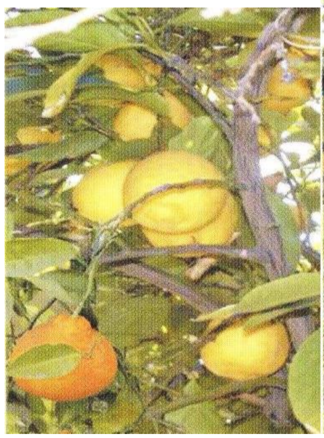


Fig. 1
Lemons, oranges, etc can grow the same tree. They belong to the same "Genus Citrus".

Fig. 2
Apples belong to "Genus Malus" and cannot grow on the same tree as lemons.

In the case of plant biology the conserved property is the Genus and the conservation law could be phrased as

"The Genus property is conserved along any connected path on any given tree"

We now know this Taxonomy is the consequence of the spatial organization of Biology's "elementary"/"fundamental" constituents, the nucleotides which are the building blocks of DNA. If we break up the DNA structure conserved properties such as "Genus" disappear. We are left with the nucleotides which are the same for all biological systems.
For biological systems the Binding Energy of nucleotides in the DNA structure are of the order of a few electronvolts (eV) which is very much smaller than the trillions of eV's which is the mass of the constituents (atoms-molecules).
In Chemistry (Atomic Physics) the classification scheme involves "Groups", "Periods", "Chemical elements" etc. We now know that Chemistry's Taxonomy is the consequence of the spatial organization of Chemistry's "elementary"/"fundamental" particles which are the electrons orbiting around the nucleus under the influence of the electromagnetic force. This was not universally accepted until the late 19th century. The debate whether chemical properties where "intrinsic" or "structural" was finally settled with the discovery of electrons and the establishment

of the Atomic Model. In fact a few centuries earlier Alchemists were trying to change the nature of chemical elements (fig.3)

Fig.3 Alchemists tried in vain to change chemical properties in order to produce gold or silver from lead or copper.

For chemical elements the Binding Energy of electrons around the nucleus are of the order of a few x10 electronVolts(eV) which is much smaller than the ~500.000 eV's which is the mass of the electrons.

In Nuclear Physics the properties of nuclei, and hence of atoms, are the result of the organization of protons and neutrons that are bound together by the strong force . The "shell model" of nuclear physics accounts for the particular combinations of protons and neutrons that make up nuclei. It is now possible to break up nuclei and thus achieve the Alchemists dream to change the nature of chemical elements.

For nuclei the Binding Energy of protons and neutrons are of the order of a few x106 electronVolts(eV) which is about 1000 times smaller than the mass of the constituent protons and neutrons (~109 eV's).

The fact that for Biological, Chemical (Atomic) and Nuclear systems the binding energy of the constituents is smaller than their mass allows us to think intuitively in terms of structures consisting of smaller entities. Furthermore the fact that the spatial dimensions of these structures range from ~ 10-9 m (nucleotides) to ~10-10 m(atoms) to ~10-15 m (nuclei) are such that they can be studied by using probes with energies that are of the same order or smaller than the mass of the constituents. This is due to the fact that in order to resolve any internal structure we must use a probe whose wavelength is smaller than the typical dimensions of the structure. The Energy-Wavelength relation

$E=hc/\lambda$, E=probe energy, λ=probe wavelength, h=Planck constant, c=velocity of light

gives us, by using the appropriate units

E(eV)=1240 x 10-9/λ(m)

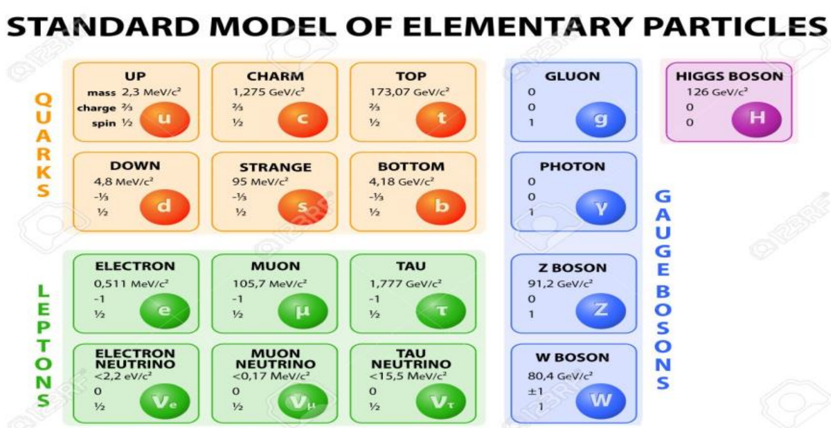
so that for λ=10-15 m we get E=1.24x1012 eV which is of the same order as the mass of the proton (~1012 eV) and for λ=10-10 m we get E=1.24x104 eV which is much smaller than the mass of the electron (~5x105 eV).

(ELEMENTARY) PARTICLE PHYSICS-THE STANDARD MODEL

The intuitive, common sense, approach to understand conserved properties in terms of an internal structure involving constituents fails us when dealing with dimensions that require probes with energies that are much larger than the masses of the presumed constituents, the reason being that the energy of the probe can create new constituents. The same is true when dealing with systems whose constituents are bound by energies which are much larger than the masses of these presumed constituents, the reason being that the binding energy can manifest itself in the form of constituents. This is the case when we go beyond Nuclear Physics and enter the world of sub Nuclear Particle Physics or simply Particle Physics.

In Particle Physics the classification scheme involves "Baryons", "Mesons" etc. In order to account for the variety, the transformations and the conserved properties of these objects in a sensible way, hypothetical constituents, called quarks, were invoked in the late 1950's. In this way the existence of various Baryons and Mesons and their properties was attributed to different combinations of quarks. For a few years, physicists argued as to whether quarks were mathematical constructs or whether they had a physical existence. In the late 1960's scattering experiments of electrons on protons, similar in many ways to Rutherford's experiment that established the presence of nuclei in atoms, revealed the presence of constituents within protons and neutrons.

These constituents were soon identified with the hypothetical quarks so that at present the so called "Standard Model" of Particle Physics involves quarks, leptons and the particles that mediate the interactions between them. The Table in Fig. 4 presents the "elementary particles" which, according to the Standard Model" account for the properties of matter in terms of constituents. Note however that these "elementary particles" are themselves characterized by certain properties (up, charm, strange, etc), some of which are conserved under transformations and whose nature is unknown.



generation I generation II generation III

Fig. 4 At least 2 of the particles in the table are familiar, electrons and photons. The table should be completed by the 12 antiparticles of the spin 1/2 particles in the table. The antiparticles have opposite electric charges.

The elementary particles in this table are organized in three generations (first three columns) consisting of particles with spin 1/2 (fermions), the fourth column contains the spin 1 particles ("Bosons") that mediate the three known interactions between the fermions , "gluons" for the "strong" interactions between quarks, photons for the electromagnetic interactions between charged particles and the W and Z particles that mediate the "weak**"** interaction responsible for some forms of radioactive decay.

The final column contains just the recently discovered, spin 0, Higgs boson. This last member is an essential ingredient of the Standard Model. It provides the Model with a mechanism that explains the fact that some of the particles have a non zero mass. Without the Higgs mechanism all the Elementary Particles of the Standard Model would have zero mass. The Higgs mechanism can account for the masses of the W and Z bosons and, with some further extensions, for the masses of the charged quarks and leptons. However, the neutral neutrinos do not acquire any mass through this mechanism.
As it is not our purpose to present here the workings of the Standard Model or the Higgs mechanism we will focus on the reasons why the model classifies the quarks and leptons in three different generations.
By way of illustration we note that for a given electric charge, say q=2/3, the three quarks, up, charm and top have different masses. The same applies to three members of the lepton family with charge q=-1, the familiar electron, the muon and the tau. If, however, masses were the only difference between the three generations one would expect that, say, the muon (μ) with a mass of ~10^8 eV should rapidly decay into an electron (e) with mass ~$5x10^5$ eV by emitting a photon (γ) as no law of nature would forbid the decay

μ==>e+γ (does not happen)

This however does not happen. Instead the muon (generation II) decays mainly into an electron (generation I), a neutrino from generation II and an antineutrino from
generation I.

μ-->e+(anti)v_e + $v_μ$

In order to account for this behaviour one attributes to members of each generation a conserved property called the lepton generation number L_I,L_{II},L_{III} (or L_e,$L_μ$,$L_τ$) with value 1 or 0 (and -1 or 0 for the corresponding antiparticles) as shown in the Table in Fig 5.

		L_e-number	$L_μ$-number	$L_τ$-number
Generation I ,	**e and v_e**	**1**	**0**	**0**
Generation II,	**μ and $v_μ$**	**0**	**1**	**0**
Generation III,	**τ and $v_τ$**	**0**	**0**	**1**

Fig 5. Lepton number assignment. For the corresponding antiparticles 1=>-1

By invoking the "conservation of Lepton number", i.e. asserting that Lepton number is a conserved property in the charged Lepton transformations one accounts for the fact that the decay μ=>e+γ is forbidden whereas μ=>e+(anti)v_e+$v_μ$ is allowed.

μ≠>e+γ ($L_μ$=1-->$L_μ$=0 and L_e=0-->L_e=1, hence forbidden)

μ=>e+(anti)v_e+$v_μ$ ($L_μ$=1-->$L_μ$=1 and L_e=0-->L_e=0, hence allowed)

The question that arises naturally with respect to this experimentally observed "conserved" property is whether it is "intrinsic" or whether it is "structural".

The "conserved" properties involving Lepton number generations have their counterparts in the three quark generations.

So far all experimental efforts to understand this "conserved" property as resulting from a composite nature of the muon have failed. The present spatial limits on internal structure are of the order of ~10-18 m corresponding to the limits of available probing energies at the CERN

LHC which are of the order of ~1015 eV. As the shortest meaningful length, the limiting distance below which we believe that the very notions of space and length cease to exist is of the order of 10-35m, the so called Planck Length, there is a long way to go, about 17 orders of magnitude, in the quest for structure of elementary particles.

In the Table of Fig.4 we also note that the neutrino masses are given as being smaller than a certain value rather than equal to 0 as predicted by the Standard Model. This arises from the experimental observation of "neutrino oscillations" i.e. the fact that neutrinos of any given generation evolve, as they travel through space, into neutrinos of a different generation. For such oscillations to take place Quantum Mechanics implies that neutrinos cannot be massless. The neutrino mass limits in Fig.5 have been deduced from the experimentally measured period of these oscillations. There are several proposed solutions to this anomaly all of which go beyond the Standard Model.

CONCLUSION

In the Standard Model of elementary particles the conserved properties, such as the Lepton generation number, attributed to the different generations of leptons are features that are introduced "by hand". They do not arise from some more fundamental theoretical considerations. In addition the behaviour of neutrinos suggests that the Standard Model is just the early version of a more complete theory.

It may be that the answer to these questions and in particular the understanding of the nature and role of neutrinos may hold the key to many other open problems, not only in Particle Physics but also in Cosmology, such as the so called Dark Matter problem and the fact that our Universe is not symmetric between matter and antimatter as expected from the Big Bang creation theories.

WORKSHOPS

MATHEMATICS AND CREATIVIY: AN APPROACH USING DIGITAL STORYTELLING

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ABSTRACT

Studying mathematics involves a scientific point of view of reality, whereas inventing and telling stories involve an empirical and imaginative point of view about it. This paper is focused on these two aspects: to combine together mathematics and creativity. The use of mathematical rules and formula and the use of storytelling allow us to think of a “different” mathematics more engaging for the students: cosine function, hyperbola, Gaussian function and Dirac Delta function are protagonists of two stories made by multimedia tools. It is a classroom activity, whose main goal is to improve mathematics knowledge based on creative and non-standard processes.

INTRODUCTION

A better understanding of the real world implies the knowledge of mathematics, without any fear or concern that it is a difficult subject. Real world means everything around us: nature, society or culture, including everyday life. The real world is complex, but a mathematical approach could help in describing this complexity. This passage from reality to mathematics and vice versa is achieved by the mathematical models and in addition to mathematical formula and rules.

What is a mathematical model? The starting point is normally a certain situation in the real world. If we are able to simplify it, to structure it and to make it more precise, according to the problem solver’s knowledge, it will lead to the formulation of a problem and to a real model of the situation (Bloom 2002):

If appropriate, real data are collected in order to provide more information about the situation at one’s disposal. If possible and adequate, this realistic model – still a part of the real world in our sense – is mathematised, that is, the objects, data, relations and conditions involved in it are translated into mathematics, resulting in a mathematical model of the original situation. Now mathematical methods come into play, and are used to derive mathematical results.

At the same time the problem solver validates the model by checking whether the problem solution obtained by interpreting the mathematical results and he decides if it is appropriate and reasonable for his purposes. If necessary, the whole process has to be repeated with a modified or a completely different model. At the end, the solution of the original real world problem is stated and communicated.

DIGITAL STORYTELLING IN MATHEMATICS

During the solution of a mathematical problem or question, we cannot conceive to use anything of different from mathematical rules or formula, nevertheless we can create a story to solve it.
While teaching, it might be that we occasionally tell a 'mathematical' story, but the stories usually concern the most famous mathematician's life or a problem formulated like a story. This is an important aspect, but it does not represent a new educational approach to solve a problem. Digital storytelling is an educational activity to make mathematics more accessible to students, as well as more engaging through.
But firstly, what is a story? A dictionary definition suggests that a story is "a factual or a fictional account of an event or series of events". Despite the feeling that "no definition is necessary", or that "every child knows what a story is", we can say that a story is a narrative unit that can fix the affective meaning of the elements that compose it (Egan 2005). Another definition is (Green 2004):

> *In essence, a narrative account requires a story that raises unanswered questions or unresolved conflicts; characters may encounter and then resolve a crisis or crises. A story line, with a beginning, middle and an end, is identifiable. […] It is generally agreed that stories are a powerful structure for organizing and transmitting information, and for creating meaning in our lives and environments.*

The common feature in both descriptions is the structure. The story must have a structure. While Green's attention is mainly on the information embedded in a story, Egan's additional focus is on the effect, on orienting feelings. Egan describes a story as a particular kind of narrative unit that orients our emotions to the events presented through the narration. That is, stories make us feel. We ascribe emotional meaning to events, and to people, and to our own lives by plotting them into partial or provisional stories. We orient ourselves emotionally to our environment by involving it in our stories. The value of the story is precisely its power to engage the students' emotions and also, connectedly, their imaginations in the material of the curriculum. In the educational activity described in this paper, the story is functional to understand geometric graphics, in doing this, it has not given precedence to student entertainment over education goals, because the main aim was to engage them. Engaging students with mathematical activity as an essential aspect of successful education. The great power of stories (Egan 2005), is in their dual mission: they communicate information in an unforgettable form and they shape the hearer's feelings about the information being communicated. These feelings can capture the attention of the pupil and involve him totally in the learning process, especially in mathematics, that is often considered as a cold discipline. Moreover, a story implies a message or a meaning as well as mathematics, which we manipulate symbols for reasoning and finding solutions.

MODELLING AND DIGITAL STORYTELLING IN MATHEMATICS. AN EXAMPLE.

In a scientific era, storytelling is considered appropriate only for language arts and advertising projects. However, in today's multimedia world, we can develop digital storytelling to practice and master a high number of skills and contents according to technological standards. The process of crafting the digital story becomes rich in skills such as: communication, collaboration, oral speaking, creativity, visual and sound literacy, project management. It also helps to develop a range of digital communication styles necessary to function in a knowledge society (Porter 2008). Two multimedia stories "The waves of the heart" and "Loving is a δ-function" are presented in this paper as an educational approach to mathematical modelling through digital storytelling.

"The waves of the heart"

Digital storytelling enables innovation and creativity. Authors (students and teachers) become creative in designing information and communicating understandings with the images, graphics, movement and music of digital media. Digital storytelling provides a unique opportunity for mixing media and mathematical contents in order to involve many pupils. This work deals with the following mathematical contents:

- Sinus and cosine graphs
- Absolute value and cosine function
- Hyperbola: definition and its properties.

The story, that we propose, has "a beginning, a middle and an end" (Green 2004).
The beginning. The protagonists are one girl (Emmy) and one boy (Stefano), they are falling in love, but later, as it often happens, Stefano breaks off their engagement and obviously Emmy is suffering.
The middle. Emmy is a good student in mathematics, therefore she begins to turn her love sickness in mathematical concepts: trigonometric function (see fig. 1) and absolute value in trigonometric function (see fig. 2) Suggest to her that love sickness will not last forever.

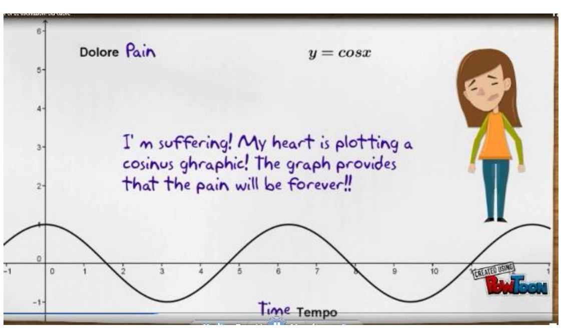

fig. 1 y = cos (x)

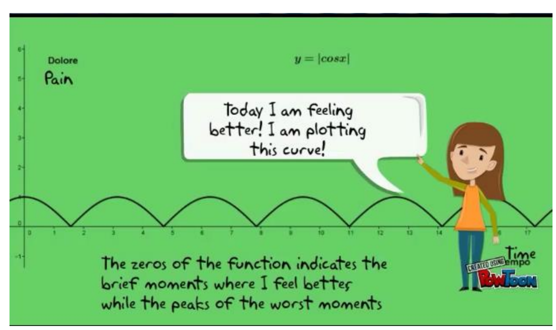

fig. 2 y = |cos (x)|

The end. Studying some properties about the cosine function and hyperbola, Emmy discovers her pain will decrease more and more, until it disappears. The other events of her life overcome her suffering, therefore she draws a new graph (see fig.3):

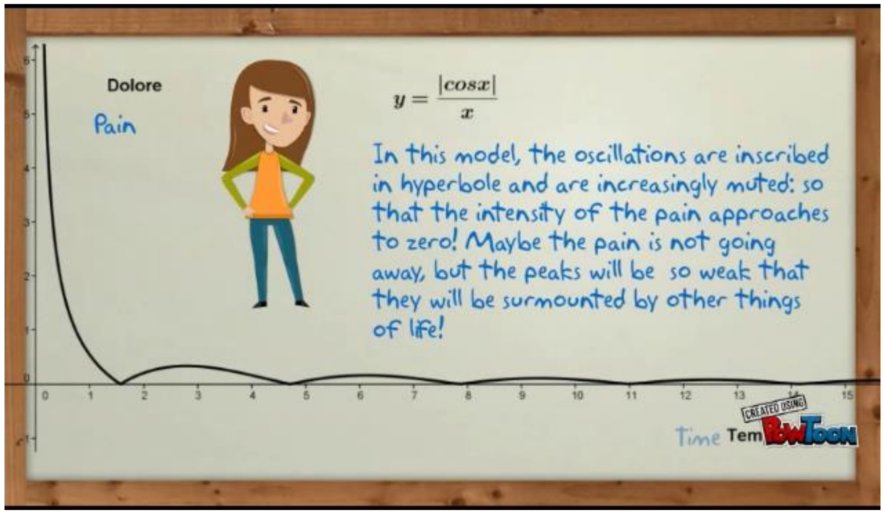

fig.3 y = |cos (x)|/x

The figures 1,2,3 are frames of a video, whose link is

"Loving is a δ-function ".

The Emmy's story goes on. It's been a while and now Emmy is less sad, one night she dreams of being a princess who dismisses all her suitors. Her sisters try to help her using Gaussian curves. But Emmy wishes to fall in love, a new great love for one boy and she uses the Dirac delta function to model her new love. In this story, the protagonists are: Gaussian function and its properties and Dirac delta function. The qr-code for looking at the video is following:

CONCLUSION

Modelling reality means applying rules and logic process of the mathematics to the real world for knowing it better. The methodology, that teachers use for it, can stave off or approach the students. Digital storytelling has an important role in the activity in order to create a “good” empathic relationship between student and mathematics.

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THE MATHEMATICS CUISINE

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ABSTRACT

Who of all of us who teach mathematics have not heard questions such as:
-Why do I need mathematics? Why do I have to learn mathematics?
Within the context of teaching mathematics in schools this subject is seen as strict and hostile by the students.
However the interactive teaching of mathematics through a plethora of games can introduce students to this “unusual world”.
Studies have shown that children who integrate mathematical games into their everyday lives develop skills such as the ability to solve practical problems, end exhibit, a better understanding of aesthetics, logic and geometry.
The “cooking” of mathematics is an attempt to show its interactive side and to prove that through the use of games it is possible to improve the relationship between children and mathematics.

An infectious disease has contaminated the atmosphere of our planet and is threatening to wipe out the human race. The scientist who has the antidote is intelligent but also forgetful. Because he was afraid he would not be able to remember the code that unlocks his lab, he has given his three colleagues some clues ... so that he can open it under any circumstances

We will need the best detectives in order to break the code. His three colleagues have in their hands a series of puzzles given to them by the scientist. But the puzzles are demanding, they require attention and thought. If you are a detective, you are invited to find the "key" that unlocks the lab so that the antidote can save humanity.

That is why you will be divided into groups. Each team will appoint a leader and all together you will name your team.

Each puzzle is rewarded with 10 points (when solved without any help).

You can ask for help, but you will be penalized by having points removed

for the 1st tip, 2 points are removed
for the 2nd tip, 3 points are removed
for the 3rd tip, 4 points are removed

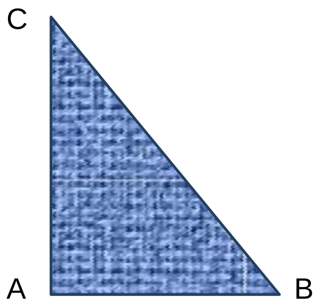
The first scientist, Mr Manolis, will give each team a card. The card has the puzzle on one side and on the other some information - instructions necessary for the solution.

When the puzzle is solved, you have to go to the next scientist, Mrs. Eliana, to get the next card. and then to Mrs. Andy to get the third one and then again to Mr.
Manolis etc. until all the puzzles are done.

The first team to find the code will receive the prize. Ready?
The leaders can come and collect the first card for their team.

Let the game begin!!!

Riddle 1:



The ABC triangle is right triangle to A with sides AB = 3, A = 4. Calculate the BC side.

If you know that the lengths of the sides AB, AC, BC are a Pythagorean triad, form another right triangle with side lengths of another Pythagorean triad.

Tips:

A right triangle is a triangle in which one angle is a right angle (that is, a 90- degree angle). The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides: (Pythagorean triad)

Example: Right Triangle with vertical sides: is equal to the sum of the squares of the other two sides: $\mathbf{z^2 = x^2+y^2}$
(Pythagorean triad)

have a hypotenuse . So 6, 8 & 10 assemble a Pythagorean triad.

How many such triads are there?

infinite triads!!!

Euclid (330-275 b.C.) gave a method of finding Pythagorean triads:

- ✓ If l, m are natural numbers and l> m then the numbers
  $x=\lambda^2-\mu^2$, $y=2\lambda\mu$, $z=\lambda^2+\mu^2$
  are a Pythagorean triad

- ✓ If x, y, z are Pythagorean triad, and k is a natural number, then
  kx, ky, kz are also a Pythagorean triad

Riddle 2:

John's age is 3 times greater than Mary's, while Steve is 2 years older than John. After 4 years, the sum of their ages will be 35. How old is each one of them today?

Tips:

A classic age problem is solved very simply if you make a table in which you place in the first column the ages of the subjects now. You have to place in the second column, the ages that these subjects will be before or after a few years. Thus an equation can be constructed which gives the solution.

See the table below

Subjects	Now	After (+) or before (-) a few years
1st subject	Age_1	Age_1
2nd subject	Age_2	Age_2

We construct the equation depending on the data we have and then we solve it.

Riddle 3:

Convert the binary number 101011 into the decimal numeral system.
Also convert the decimal number 47 into the binary numeral system.

Tips:

To convert a binary number into a decimal, we need to multiply every binary number with the corresponding power of by starting from the least significant digit (the 1st from right to left) which represents the exponent 0 of the power of 2 (. Moving forward to the digits, each exponent is increased by 1. Add all the products that result and their sum is the decimal number.

Example:

$(1001)_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 0 + 0 + 1 = (9)_{10}$

To convert a decimal number into the binary numeral system we use the following procedure:
1) We divide the number by 2.
2) We continue the divisions with 2 of the individual quotients, until a zero quotient is found.
3) The individual remainders that emerged are placed in the row by writing each digit from right to left (reverse writing). That means that the first remainder will be placed as the last digit of the binary number and the last remainder will be placed as the first.

Example: We want to convert number 16 into the binary system.

16 : 2 = 8 quotient and 0 remainder
8 : 2 = 4 quotient and 0 remainder
4 : 2 = 2 quotient and 0 remainder
2 : 2 = 1 quotient and 0 remainder
1 : 2 = 0 quotient and 1 remainder

So the binary number is: 10000

Riddle 4:

Factorize the trinomial: $P(x) = x^2 - 7x + 12$
Calculate the number
$24^2 – 7 * 24 + 12$using the previous result.

Tips:

- The trinomial form is $ax^2 + bx + c$
- It's discriminate is: $\Delta = b^2 – 4ac$

$$if\ \Delta > 0 \underset{then}{\Longrightarrow} x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

When x1,2 is known, then i can factorize the trionymous into:
$ax^2 + bx + c = (x – x_1) * (x – x_2)$

Riddle 5:

A teacher splits his students into groups of 3 but there are two left over Then he decides to split the students into groups of 4 but, again, there are two left over.
Finally he decides to split them into groups of 5 and there are none left over. How many students does this teacher have?

Tips:

1. Multiples of an integer value a, are the products that result if we multiply a with all integer values
   e.g. 0 . a, 1 . a, 2 . a, 3 . a, 4 . a etc

2. Every integer value can divide all of its multiples

3. If an integer value a is divided by an integer value b
   then it is a multiple of b.

The digits of the Lab's key are:

1st and 2nd digit: The number of kids from the 5th riddle

3rd digit: John's age in the 2nd riddle

4th and 5th digit: The (101011) number to decimal form in the 3rd riddle

6th and 7th digit: The 2 first digits from the result in the 4th riddle

8th digit: Hypotenuse's side length in the 5th riddle

9th digit: The first letter of the female name in the 2nd riddle

Lab's Key: _ _ _ _ _ _ _ _ _

(50 9 43 4 2 M)

CONGRATULATIONS
the humanity will be saved.
the team .... found the code and took the prize which is a great chocolate.

SOLUTIONS

Riddle 1:

According to Pythagorean Theorem:
$AB^2 + AC^2 = BC^2 \Leftrightarrow 3^2 + 4^2 = BC^2 \Leftrightarrow BC^2 = 25 \Leftrightarrow BC = \sqrt{25} = 5$

Thus the number 3, 4, 5 consist of a Pythagorean triad.

If l = 5 & m then

$x = l^2 – m^2 = 25 – 16 = 9$
$y = 2 * l * m = 2 * 5 * 4 = 40$
$z = l^2 + m^2 = 25 +16 = 41$

Thus the numbers: 9, 40, 41 consist of a Pythagorean triad

Riddle 2:

Subjects	Now	After 3 years
John	$3 . x$	$3 . x + 4$
Mary	X	$X + 4$
Steve	$3 . x + 2$	$(3 . x +2) + 4 = 3x +6$

Thus: 3x + 4 + x +4 + 3x + 6 = 35 ⇔
7x + 14 = 35 ⇔
7x = 21 ⇔
X = 3

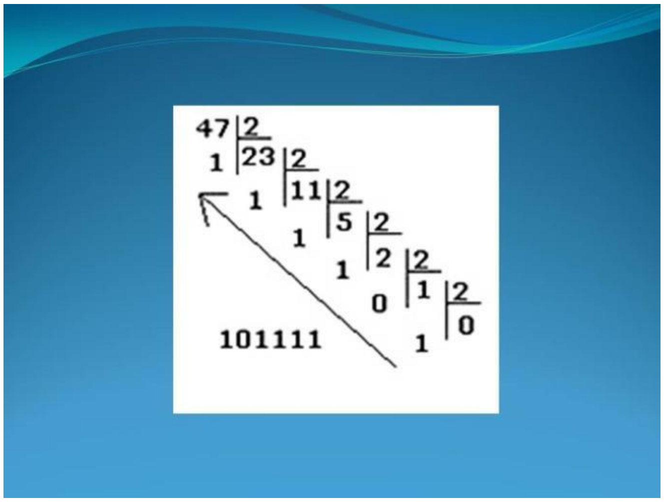
So today Mary is 3 years old, John is 9 years old and Steve is 11 years old.

Riddle 3:

binary ⟶ decimal

$101011 = 1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 2^0 = 32 + 0 + 8 + 0 + 2 + 1 = 43$

decimal ⟶ binary



Riddle 4:

Roots of the trinomial: 3, 4 so

$P(x) = (x – 3) . (x – 4)$

We observe that the number

$24^2 - 7 * 24 + 12$

P(24)=(24−3) (24−4) = 21 20 = 20 20+1 20=420

Riddle 5:

If the number of children is **x** then **x-2** is a multiple of 3.
Also **x-2** is a multiple of 4.
The integer multiples of 12 are: 12, 24, 36, 48, 60, 72 etc.
But **x** is a multiple of 5.
So if we add up to the previous multiples the number 2 we need to get a multiple of 5.

Thus the answer is 48 + 2 = 50
So x=50

When does the game end and begin serious math? For many, maths is considered a deadly boring subject which has nothing to do with gaming. It is a set of acts, rules, theorems strictly described without any relationship to games and pleasure.

On the contrary, in the eyes of most mathematicians it never ceases to be a game, but it is also many other things.

Today we tried to give a sample of our "kitchen", so we dealt with:

- the Pythagorean theorem and the Pythagorean triads.
- problems solved with equations,
- the binary and decimal numbering system,
- trinomials
- multiple numbers,

Is it possible that playing games is actually mathematics?

LEARNING MATHEMATICS WITH RUBIK'S CUBE

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ABSTRACT

When you have a first contact with the Rubik's Cube, You admire its shape, color patterns and eventually you start moving its faces. The color pattern changes with each rotation, from the solved state to the scrambled state where many stickers are mismatched. Couple moves later the cube is now really messed up and that is the right moment to start thinking about the inverse problem – given a scrambled cube you want to restore its original pattern, where each face has only one color of the stickers. In this article we present a journey via some mathematical aspects of the cube and explain how to solve it by using some abstract yet intuitive ideas.

1. INTRODUCTION

The Rubik's Cube was invented by Erno Rubik [2], a Hungarian inventor and architect. His adventure with creation of the puzzle finished in 1974, when he completed his first wooden version of the cube. Three years later the cube was released in the Budapest; it took three more years to release the cube to the world. It quickly became one of the best-selling puzzle at that time, and it holds that title up to present day with more than 300 million units.

Astonishing fact about the cube is that it took its inventor about a month to solve it. Today, we have many websites offering guides and tutorials [1], both video presentation and spreadsheets with algorithms. They offer set of moves that one has to follow to complete the puzzle. While that is an "easy" way to complete the cube, it does not carry any logic behind the actual solution. By the logic we understand the following problem: how to figure out the actual sequence of moves?

In this article we provide a simple mathematical approach to the problem of solving Rubik's Cube. We introduce some simple algebraic notions that are necessary to fully understand the process of creating the solution. Thus, we divide the article into several sections. Firstly, we provide a background theory of groups and (non-)commutative actions. Then we describe the Rubik's Cube as a group of a special kind of actions – permutations. We use notions of conjugacy and commutator to explain the idea behind some sequences of moves. Finally, we present the complete solution of the puzzle.

Throughout the article we use some convenient name for different pieces of the cube: we call each of the 26 composing cubes "cubies", a cubie with 2 stickers an edge and a cubie with 3 stickers a corner. Recall that if a fixed orientation of the cube is set (that is: we only rotate faces so that the position between center pieces does not change), the middle stickers define the color of the face at solved state as well as these cubies cannot swap between one another.

2. SOME ASPECTS OF GROUP THEORY

The Rubik's Cube can be seen as a group with a very specific action.
Definition 2.1. Consider any set G and an algebraic operation $*$. We call a pair $(G,*)$ a *group* if the following set of conditions is satisfied:

- $a * b \in G$ for any $a, b \in G$ (well-defined operation),
- $(a * b) * c = a * (b * c)$ for any $a, b, c \in G$ (associative operation),
- there exits an element $e \in G$ such that $e * a = a * e = a$ for any $a \in G$ (neutral element property),
- for any $a \in G$ there is $\#a \in G$ such that $a * \#a = \#a * a = e$ (inverse element property).

It is casual to denote the inverse element by a^{-1}. We call a group $(G,*)$ commutative (abelian) if additionally

- $a * b = b * a$ for any $a, b \in G$.

Example 2.2. The following pairs are groups: $(\mathbb{Z}, +), (\mathbb{Q}\backslash\{0\},\times)$ (the cross stands for multiplication sign), the set of all polynomials with the " $+$ " operation, the set of all congruences on the plane (non-commutative), permutation group of the Rubik's Cube (non-commutative as well). The pairs $(\mathbb{Z},\times)$ and $(\mathbb{R},\times)$ are not groups.
We leave the proof of the above statement as a simple exercise for the Reader.
We now describe the group of permutation. Consider any finite set A.

Definition 2.3. A permutation is arbitrary arrangement of the elements of A. A group of permutations of A is a collection of all permutations of A with the operation of composition of permutations.
We explain the above definition in the following example.
Example 2.4. Let $A = \{1,2,3,4\}$. A permutation changes the order of the elements by defining the outcome for each element of A. Take for instance the following permutation: $4, 3, 2, 1$. In this permutation first element is on the fourth place, second is on third etc. This can be represented in the arrow notation as follows (the left column is the number and the right column is the position it is mapped to):

$$1 \to 4$$
$$2 \to 3$$
$$3 \to 2$$
$$4 \to 1,$$

We can briefly describe that permutation using the bracket-cycle notation:

$$(1\ 4)(2\ 3),$$

which is understood in the following way: we look at each sequence in the bracket and follow the path from left to right; if there is no number to follow, we cycle back to the first number. The cycle is a single bracket described as above. Consider two more examples:

$$1 \to 3$$
$$2 \to 4$$
$$3 \to 2$$
$$4 \to 1,$$

briefly: $(1\ 3\ 2\ 4)$;

The second example is the following:

$$1 \to 2$$
$$2 \to 3$$
$$3 \to 1,$$

briefly: $(1\ 2\ 3)$.

Note that we skip 4 in the second example – if the number is not permutated we do not include it in the bracket-cycle notation. We suggest, as an exercise, to decode two permutations to the arrow notation: $(1\ 3)(2\ 4)$, $(4\ 2\ 3)$.
We now describe how to compose two permutations. Consider two permutations $(1\ 2\ 3\ 4)$ and $(1\ 2\ 3)$. In order to compose them, that is to find the permutation

$$(1\ 2\ 3\ 4)(1\ 2\ 3),$$

we follow the change in values from right to left: 1 goes to 2 (the right permutation) and 2 goes to 3 (the left permutation). Using the arrow notation we can write it as follows:

$$1 \to 2 \to 3,$$
$$2 \to 3 \to 4,$$
$$3 \to 1 \to 2,$$
$$4 \to 4 \to 1.$$

Briefly: $(1\ 3\ 2\ 4)$.

We can check that in such a case the set of all permutations of the set A is a group with the operation of composition. In particular, each permutation has its inverse one and the neutral element of such an operation is the identity permutation, that is the arrangement $1, 2, 3, 4$ for the case of the above example.
We now need to establish the notion of disjoint cycles.

Definition 2.5. Two cycles are disjoint if no common number appears in their description.
One can think of that property as some sort of the property of the entire permutation. The permutation itself can be a composition of several cycles (the first permutation described in Example 2.4 is such) or a single cycle. Cycles that are disjoint cannot be composed to form a new cycle consisting of all elements of initial cycles, while the ones that are not disjoint can (like it was for the case $(1\ 2\ 3\ 4)(1\ 2\ 3) = (1\ 3\ 2\ 4)$).

Remark 2.6. Any permutation can be decomposed into the composition of disjoint cycles.

Definition 2.7. The order of the cycle is the number of elements in the cycle. The order of the permutation is the least common multiplier of orders of composing and disjoint cycles.
One can also think of the order as the number of applications of the same permutations to obtain the identity permutation.

Example 2.8. The permutation $(1\ 2\ 3\ 4)(1\ 2\ 3) = (1\ 3\ 2\ 4)$ has order 4. The permutation $(1\ 4)(2\ 3)$ has order 2.
We leave the following as an exercise: evaluate the order of the following permutation:

$$(1\ 3)(2\ 4)(2\ 3)(1\ 3).$$

Definition 2.9. Consider two elements a and b of the group $(G,*)$. The commutator of them is the element $[a\ b]$ defined by the relation

$$[a, b] = a * b * a^{-1} * b^{-1}.$$

Definition 2.10. We call two elements a and b of the group $(G,*)$ conjugated (and their respective relation conjugacy) if there exists an element $g \in G$ such that

$$g * a * g^{-1} = b.$$

In that case we also say that a is conjugated to b and vice versa.

3. DESCRIPTION OF THE RUBIK'S CUBE

Each cubie of the Rubik's Cube can be described using two or three letters from the set $\{R, F, U, L, B, D\}$ – each letter corresponds to one of 6 faces: [R]ight, [F]ront, [U]p, [L]eft, [B]ack, [D]own. In order to describe the cube using permutation we fix an orientation of it: for instance the middle sticker on the front face can be red and the middle sticker on the up face can be green. Then we associate with each cubie 2 (the edge piece) or 3 letters (the corner piece). For instance,

- RF denotes the cubie that is on the R face as well as on the F face (in our fixed orientation it is white-red edge),
- RFU denotes the cubie that is on R, F and U faces (in our fixed orientation it is white-red-green corner).

Note that the order of letters matters – FR describes the same edge, but with different order of colors.
We can now describe a permutation of elements of the cube when the right face is turned 90 degrees clockwise. The resulting permutation is the following pattern

$$(FRU\ URB\ BRD\ DRF)(FR\ UR\ BR\ DR),$$

which in particular is composition of two cycles of order 4, hence the entire permutation is of order 4 and 4 rotations of the right face restore the original permutation of the cube. That agrees with the intuition built without the mathematical background.
Let us also highlight the fact that one can actually choose FUR as first element of the cycle, and then the following ones would change accordingly – this describes the permutation of the same pieces of the cube, but with different stickers order. We chose one, but the Reader can choose

any other – the resulting permutation will slightly differ – the difference would be the order of letters in following elements, but not the elements itself.
Remark 3.1. Every arrangement of the cube can be described as some permutation.

Example 3.2. If we rotate the right face clockwise and then top face counter-clockwise, both 90 degrees, the resulting permutation is

$$(FR\ UB\ UL\ UF\ UR\ BR\ DR)(FUR\ ULB\ UFL\ URF\ LBU\ FLU\ FRU\ BUL\ LUF)$$
$$(FDR\ URB\ BRD\ DRF\ RBU\ RDB\ RFD\ BUR\ DBR).$$

The permutation consists of three disjoint cycles of orders $7, 9, 9,$ respectively. Hence, the entire permutation is of order 63. We suggest checking with the actual cube rotations that 63 alternated rotations of the right and the top face in a clockwise direction restore the cube to the solved state.
We propose the Reader to describe a rotation of the right face 90 degrees clockwise followed by a rotation of the top faces 90 degrees clockwise.
We can now see that the Rubik's Cube is actually a group of permutations with the operation of composition of permutations. This allows us to use the group theory, especially the notions of conjugacy and commutator to find a solution to the puzzle.

4. SOLUTION

In this Section we use basic principles and ideas presented previously to provide a set of algorithms that allows to solve the cube.
The first step is to introduce a convenient notation for rations of each face or slide of the cube. We use:

- R, U, F, L, B, D for a 90 degrees rotation of right, up, front, left, bottom and down face, respectively,
- M, S, E for a 90 degree rotation of the slice between R and L (according to L), F and B (according to F) and U and D (according to D), respectively.
- if the same move X is applied twice in a row, we shorten XX to $X2$,
- we denote the inverse move to X by X'.

Remark 4.1. We have the following:

- $(X2)' = X2,$
- $(XY)' = Y'X'$ (this is the general rule for non-commutative groups).

There are more than 42 quintillion scrambles of the cube. To solve them all, it is sufficient to use the specific set of algorithms that we describe and explain below.

Orientation of two edges: let

- $X = L\ E\ L2\ E2\ L$,
- $Y = U$.

Then

$$[X,Y] = (L\ E\ L2\ E2\ L)U\ (L'E2\ L2\ E'L')U'$$

is a commutator that swaps the orientation (but not the position) of UL and UF edges (the notation is according to the one introduced in Section 3).

The algorithm works as follows: the X move flips the orientation of the UL edge while keeping the rest of upper face intact. By performing the Y move we change the edge that is now affected by the X' move, that undo the scrambled part of the cube while flipping the UR edge. The entire algorithm therefore flips two adjacent edges on the top face (changes their orientation).

This algorithm is of order 2.

Permutation of three edges: let

- $X = M'$,
- $Y = U2$.

Then

$$[X,Y] = M'\ U2\ M\ U2$$

is a commutator that permutes FD, FU and UB edges (it preserves their orientation).

The breakdown of this algorithm is simple: the $M'U2$ sequence swaps 3 edges in the middle slice, then the M move restores centers to their original position. Finally, the $U2$ move fixes the positioning of most top pieces.

Since three pieces are affected, the permutation has order 3.

Orientation of two corners: let

- $X = F'D\ F\ L\ D\ L'$,
- $Y = U$.

Then

$$[X,Y] = (F'D\ F\ L\ D\ L')\ U\ (L\ D'\ L'\ F'\ D'\ F)\ U'$$

is a commutator that rotates FUR corner clockwise and UFL corner counter-clockwise.

This slightly harder algorithm rotates the FUR corner first while keeping the rest of the upper layer intact (the X move). Then we proceed similar to edge orientation – we place other corner in place of FUR one and reverse the X move.

Since two pieces having three possible states are affected, the order of this permutation is 3.

Permutation of three corners: let

- $X = F\ L\ F'$,
- $Y = R2$,
- $Z = F2$.

Then

$$Z\ [X,Y]\ Z' = F2\ (F\ L\ F'R2\ F\ L'\ F'\ R2)\ F2$$

Is a conjugacy that permutes ULF, URF and URB corners.

The $F2$ moves are simply setup moves for the actual commutator. Unlike in other cases, this algorithm has no simple intuition.

This permutation has order 3.

In order to solve the Rubik's cube using these algorithms we first solve edges, then corners (or corners first – it does not matter, but we find edges easier as first). Note that in order to solve the edges one has to first permute the pieces, then to orient them. In order to permute them,

one has to make a setup moves so that all pieces that are affected by the commutator are in correct places. The same principle follows for the permutation of edges. For orientation, careful solution can reduce the number of setup moves to $2-3$.
Once the setup moves are performed, we can apply the algorithm and then undo the setup moves, but we have to remember that they have to be done in reverse order and each move must be an inverse as well. For example, if the setup moves where $R\ F\ U$, then to undo them we do $U'\ F'\ R'$.
One can think of the above idea as the conjugacy – setup moves are followed by the actual algorithm which is followed by the inversed setup moves.

5. FINAL REMARKS

There are many other algorithms that can be used to solve the cube, like for instance the sequence

$$(M'U'M'U'M'U'M')\ U\ (M\ U\ M\ U\ M\ U\ M)U'$$

is a different pattern of moves that swaps the orientation of two edges. In fact, the basic principle of most algorithms that we have presented or one could search for is usually based on the two notions: conjugacy and commutator. They allow to manipulate the cubies in a specific and controlled way. Such a solution is also a good approach not only for a person willing to find out the solution on their own, but also for mathematicians.

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